Optimizing Open Pit Limits Without and With Ore Dressing Predictions

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Abstract
One of the crucial problems in open pit optimization is the correct definition of the ultimate ore body shape with economically recoverable reserves. A key factor consists of simulating the mineral liberation phenomena and predicting ore dressing parameters in borehole samples. Such attribute information is essential for construction of the economic block model of a deposit. In this case optimal pit shells can be generated more reliably.

This paper outlines items inherent in modeling and simulating low-grade iron ore bodies to find optimized mining sequences for long term planning using a generalized mining program with an interface to the Whittle software.

Introduction
The estimation of recoverable ore reserves, open pit optimization and mine planning are interdependent procedures linked by a set of parameters which should be defined in detailed mathematical model of a deposit. Consequently from starting point of sampling, it is important to outline external boundaries of ore bodies and taking in consideration a variety of different types of information, then proceed to construct mineralurgical and economical models of the deposit (Whittle and Vassiliev, 1997). In some cases ore characterizations may be limited to a simple definition of metal concentrations with constant mining and metallurgical recoveries inside entire model, but as a rule for multi-element complex rocks with a high degree of spatial variation, the impact of petrology and textures on subsequent mineral processing are very essential (Clout, 1998). Since it is usually impractical and too costly to testwork all geological samples under various ore-dressing and extraction options, the main input parameters, such as grade, yield and recovery, must be predicted with simulation techniques for more proper cut-off optimisation.

Progress in quantitative mineralogical image analysis and achievements in the comprehension of mineral liberation phenomena (King, 1979; Barbery, 1988; King, 1998; Vassiliev and Tikhonov, 1988; etc.) assist in development of new approaches in ore-dressing predictions with more effective techniques than those based only upon statistical correlation of testwork data and ore properties. Because changes in processing cost, in which the lion’s share consists in crushing and grinding operations, to obtain acceptable mineral liberation degrees, have a direct effect on the determination of economic cut-offs so applied results in finding correct optimal pits and improvements in mine planning are expected.

This paper investigates a methodology for predicting mineral liberation and separation parameters for iron ores in ferruginous quartzite deposits, taconites, while using open pit optimizers like Three-D and Four-D packages from Whittle Programming Pty Ltd.
Parameters

The recoverable reserves of a deposit apart from other factors are dependent on the size of selective mining units (SMU). The size of the SMU generally depends on many factors, mainly the capacity and size of loading equipment, the bench height, the variability of ore petrology and the parameters of blasting. In fact you should know not only integral grade and tonnage of SMU in digging polygons after sampling of blast holes, but also variations of other properties like modal mineralogy, microtexture, fractures, geomechanical properties and so on, inside the excavator bucket to help in distinguishing between waste and ore. This is the complex task with a great portion of uncertainty for a big size of SMU, but if we mentally reduce the bucket to a size of several microns, suitable to mineral grains, then such equipment can mine reserves of the valuable mineral with minimum losses, dilutions and inefficient productivity. A similar exercise could be applicable for size reduction of monolithic blocks in computer regular block model of a particular deposit to show one more time the influence of dimensions.

Let us consider a selective mining unit (SMU) of ore with magnetite as valuable mineral. In order to predict principal integral volumetric mineral dressing parameters, such as $\gamma_c$ – the yield of concentrate, $\beta_c$ – the magnetite grade of concentrate and $\varepsilon_c$ – the recovery of magnetite in concentrate then we can use the following expressions (e.g. in Vassiliev and Tikhonov, 1988):

Equation 1:

$$\gamma_c = \sum_{i=1}^{n} \sum_{j=1}^{m} c_{i,j} \gamma_{i,j}$$

$$\beta_c = \gamma_c \sum_{i=1}^{n} \sum_{j=1}^{m} \beta_{j} \gamma_{i,j}$$

$$\varepsilon_c = \beta_c \gamma_c / \alpha_o$$

where:

- $i=1..n$ is the index of size class;
- $j=1..m$ is the index of grade class;
- $\varepsilon$ is the probability function of recoveries to the concentrate in grade classes;
- $\gamma$ is the size-grade matrix of particles or blocks after grinding material in SMU;
- $\beta$ is the vector of volumetric contents of magnetite in classes of grade scale;
- $\alpha_o$ is the mean ore grade in the concentrator feed.

It is known that in ball milling practice usually because of existence of locked particles and fine fractions in comminuted products, it is practically impossible to separate particles of mixtures on concentrate and tailings with a fixed cut-offs even if the gangue and mineral of interest have highly contrasting physical or chemical properties. Therefore the process of recovery in concentrate should be adopted as not perfect or not ideal separation. For the whole ore-dressing process we have to pursue the aim to connect the optimization with such expedient economic criteria as maximum profit in cash flow $P_c (\$/h)$:

Equation 2:

$$P_c = Q \gamma_c f_p (\beta_c) - C_p \rightarrow \max,$$

where:

- $Q$ the productivity in tons per hour;
- $f_p (\beta_c)$ the function of price for a ton of concentrate defined by grade $\beta_c$;
- $C_p$ the cost of processing of a ton of ore, including mining works.

Some advanced aspects of concentrator optimization problems in terms of energy efficiency, concentrator strategy and operating philosophy can be found in Morrison (1993).

In common case to forecast the size-grade matrix $\gamma$ of ore fragments after crushing and
grinding operations we should have some quantitative characteristics of micro and mesotextures inside indestructible blocks before size reduction. The examination of microscopic images of a polished sample can lead to determination of numerous mineralogical, spatial, geometric, and morphological parameters with mineral grains differing by its chemical and physical properties. However for final quantification of microtextures and mesotextures from the position of further utilization of ore reserves like comminuted materials the liberation spectrum of target mineral should serve as the very exact and exhaustive characteristic. Each mineral has its own liberation spectrum for a core sample, block or entire deposit. Figure 1 shows magnetite liberation spectra for low-grade iron ore of a taconite deposit.

Magnetite Liberation in Iron Deposit

![Graph showing magnetite liberation in iron deposit](image)

Figure 1: Liberation Spectra for Block Model with Different Block Sizes

Mineral liberation spectra are defined for random position of blocks relatively to waste-ore or magnetite-gangue boundaries. In excavation range the mining technology should aim to improve opening along ores and waste contacts and inside grinding range the ore-dressing technology must promote to liberate valuable and gangue minerals along their micro contacts. The latter is not performed in practice for hard rocks and it is possible to use a random model pattern for predicting the size-grade particle distribution $\gamma$ from the mineral liberation matrix $L$. For homogeneous multi-mineral
ores the next equation can be written in matrix form:

**Equation 3:**

\[
\tilde{a} = L \cdot f
\]

where \( f \) is the expected particle size distribution density for SMU material.

In order to simulate the distribution \( \gamma \) from textures of heterogeneous ores we should take into account some additional factors like mineral microhardness and strength of intergrowth contacts which have direct effects upon grindability and selectivity during ore comminution inside real equipment. One of the ways to simulate kinetics of size reduction with liberation process in multi-mineral heterogeneous ores consists of an adaptation of Markov chain analysis for decision of the problem. Figure 2 illustrates the concept of resorting grades in simple free and composite blocks for two adjacent size classes under reduction.

![Diagram](image)

**Figure 2: Links for Resorting of Free and Composite Blocks Under Random Size Reduction**

Accordingly to this scheme the transition probability matrix \( M \) for two-phase ore with gangue and target minerals in blocks is represented in Table 1.

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**Table 1: The Transition Probability Matrix for Grade Resorting of Blocks**

The Markov process for liberation of magnetite ores has two absorbing states in the first and end classes of grades. To archive more adequate simulation it was proposed to use a diagonal matrices \( A \) for the speed of grade resorting in relation with specific mineral surfaces and \( H \) for the selection of breakage accordingly to known mineral hardness. The probabilities of breakage \( U \) with respect to strength of intergranular contacts may be assigned for simple locked blocks to change transitions in \( M \).
In disseminated ores the values of matrix $\Lambda$ for each class of sizes can be calculated by formula:

**Equation 4:**
$$\Lambda_i = \frac{\bar{l}_{\text{target}} + \bar{l}_{\text{gangue}}}{2 \bar{l}_{\text{particle}}}$$

where round brackets embrace mean chords of mineral phases defined with the help of linear stereological analysis in corresponding $i$ class of particles. Alternatively the chords can be calculated simultaneously with $L$ while examining ore microtextures. So the values of $\Lambda$ changes from very small quantities in coarse classes to one in fine classes of volumes.

Micro hardness values in the diagonal matrix $H$ might be described as:

**Equation 5:**
$$H_j = \frac{h_{\text{gangue}} \cdot g_j + h_{\text{target}} \cdot (1 - g_j)}{h_{\text{target}} + h_{\text{gangue}}}$$

where

$h_{\text{target}}$ and $h_{\text{gangue}}$ are micro hardnesses of target and gangue minerals;

$g_j$ is the mean grade in grade class $j$.

Omitting interim computations, the matrix equation for size reduction/liberation process defined on a texture of the heterogeneous ore can be written in the next form:

**Equation 6:**
$$= Lt \cdot \prod_{k=1}^{t} \left( [MH \cdot I_m - H] \cdot [BS + I_n - S] \right)$$

where:

$k = 1..t$ is the index of time period;

$n$ is the maximum number of size classes;

$m$ is the maximum number of grade classes;

$t$ is the maximum number of size reduction circuits;

$M$ is the Markov transition probability matrix for resorting of grades, $m \times m$;

$H$ is the function to take into account the individual mineral hardness, $m \times m$;

$B$ is the function of Broadbent-Callcott; 

$S$ is the diagonal matrix for speed of breakage, $n \times n$;

$\Lambda$ is the diagonal matrix for selection to mineral grade resorting with respect in specific surfaces of mineral phases contained in ore particles, $n \times n$.

**Measurements and Modeling**

In the work for modeling of low grade iron ore deposit and optimizing open pit limits the appropriate methodology for ore-dressing prediction has been developed. Overwhelming number of samples were tested with classical chemical and mineralogical methods. And a lot of samples were investigated particularly using the IBAS (Optron, Germany) image analyzer. Figure 3 shows simplified scheme with main paths and operations from sampling up to final open pit optimization that were tested for Stoilenskoye iron ore deposit.
Figure 3: Scheme of Mineralurgical Orebody Modelling and Open Pit Optimisation

As was emphasized in Lyman etc. (1996), the new methods of quantification and modeling of ore textures must open up new dimensions in ore mineralogy and texture research, and enable major advances towards the ultimate goal of quantitative mine planning and recovery modeling. Powerful image-analysis systems have been developed in recent times specifically tailored to the Mineral Liberation Analysis (MLA). For example in Comminution Center of Utah University (USA) and Julius Kruttschnitt Mineral Research Centre (Australia). Images can be generated by scanning electron microscopy or optical microscopy and every image can have up to 2048 x 2048 pixels. A complete range of image analysis and stereological correction procedures have been developed including many that are designed specifically for mineral liberation studies (King, 1998; Gay and Wei, 1998).

The technique of modeling for Stoilenskyi iron ore deposit and ore-dressing predictions from texture simulation was constrained with one dimensional representation of phases. More fruitful but complex and prolonged method should perform synthesis of real 3D ore textures with determination of the mineral liberation
matrices and expected results of its interaction with different fracture patterns.

**Algorithm and Software**

The general procedure for one dimensional representation of volumetric texture and fracture pattern is as follows:

1. Generate a series of mineral and gangue linear intercepts according to their volume phase distributions. These are generated randomly by turns for the mineral phases along a line that should contain many thousands of simulated intercepts.

2. For each size in the expected particle size distribution, dissect the line of simulated intercepts by the particle size. This will create a population of particles for each particle size with differing liberation spectrum of a target mineral.

3. Multiply each of the spectrum produced in step 2 by the percentage yield for the corresponding particle size in the expected Particle Size Distribution. The combined populations now contain a weighted set of particles in a range of sizes and a range of grades.

4. The recovery probability factors (from the generalized Recovery Probability Distribution) are applied to the particles to determine the distribution of particles after separation.

5. Calculate the grade, yield and recovery from the distribution produced in step 4 thus predicting main ore-dressing parameters.

The algorithm represents a volume size distribution of minerals in ore texture as a sequence of interconnected line segments. The mean length of the segments corresponds to the mean chord of mineral phase in the ore sample. Subsequently the homogeneous or heterogeneous fracture patterns could be simulated as an overlay to the texture to calculate mineral liberation spectra with or without such property as mineral hardness. Taking into account the breakage, classification and separation matrix for a generalized mineral processing scheme it can be possible to predict yield, grade and recovery in a concentrate for every target mineral under consideration.

A one-dimensional model for mineral liberation is based on the idea that two-phase ore may be adequately characterized by a conformable phase parameter, tightly dependent upon the mineral aggregate size distribution. For mineral dressing goals a multi-phase ore can be consequently analyzed as a series of two-phase ores. In a two-phase ore we have the waste phase A and the target valuable phase B. Transition from metal to volume mineral assays for biphasic texture are:

**Equation 7:**

\[
\text{ASSAY}_B = \frac{(\text{METAL\_GRADE}_B/\text{B\_DENSITY})}{((\text{METAL\_GRADE}_B/\text{B\_DENSITY})+\text{METAL\_IN}_B - \text{METAL\_GRADE}_B)/\text{A\_DENSITY}),
\]

where:

- **METAL\_GRADE** – the weight per cent of a target metal in crude ore.
- **METAL\_IN\_B** – the limit weight per cent of metal from formular in mineral B.
- **A\_DENSITY** and **B\_DENSITY** – densities of minerals from a reference book.

It is important to emphasize that for a full range of mineral dressing simulations, one needs to know the natural sizes of mineral grains in the unbroken or only coarsely broken ore, more exactly the natural sizes of mineral grain aggregates. Linear, areal and volume content of a mineral phase in representative polished sections of a sample in respect with stereological principles are equivalent. It is exactly true without any assumption of size or shape of mineral grains.

The first parameter needed is the volume content Assay of the target mineral phase B in the every sample or block. The second parameter needed is the specific mean aggregate length, of the Grain of valuable mineral phase B interceptions, measured by scanlines passing over grains within the
polished samples. From input table user should select in dialogs:

- a Field to load Assay as the content of a target mineral phase
- a Field to load Grain as the size of target mineral phase
- an Expected Size Distribution of particles after grinding
- an Expected Recovery Probabilities of particles in a concentrator

After calculating in the table there should be created additional fields:

GRADE_SelectedMineral, % Volume
YIELD_SelectedMineral, % Volume
RECOVERY_SelectedMineral, % Volume

At this stage we need to compare between testwork data under usually restricted set of technological samples and the forecast data. The testwork samples are gages points and we can use them to correct expected recoveries and fit with the real experimental results. Such fitting transforms the entire data set to actual recoveries, but with the proposed scheme we have an opportunity to compute potential recoveries for any mineral phase in ores under consideration.

The target mineral phase may consists of one or several different minerals. If we need to get recovery for a target metal then we must calculate it from the mineral phase using formular metal content and density of the mineral. On the other hand, several mineral phases sometimes may contain a target metal. In such cases it is necessary to predict recoveries for every mineral phase and then calculate the total recovery of the target metal from the sample.

If Fe\_IN\_B is the concentration of target metal Fe in B in weighted per cents, then:

Equation 8:

\[
\text{GRADE\_METAL} = \text{METAL\_IN\_B} \times \text{GRADE\_B} \\
\text{DENSITY\_B} \times \text{DENSITY\_CONCENTRATE} \\
\text{YIELD\_METAL} = \text{YIELD\_B} \times \text{DENSITY\_CONCENTRATE} / \\
\text{YIELD\_B} \times \text{DENSITY\_CONCENTRATE} + (100 - \text{YIELD\_B} \times \text{DENSITY\_TAILINGS}) \\
\text{RECOVERY\_METAL} = 100 \times \text{GRADE\_METAL} \times \text{YIELD \_METAL/ASSAY\_METAL;}
\]

Where:

Equation 9:

\[
\text{DENSITY\_ORE} = \text{ASSAY\_B} \times \text{DENSITY\_B} + (1 - \text{ASSAY\_B}) \\
\times \text{DENSITY\_A}; \text{DENSITY\_CONCENTRATE} = \\
\text{GRADE\_B} \times \text{DENSITY\_B} + (1 - \text{GRADE\_B}) \times \text{DENSITY\_A}; \\
\text{DENSITY\_TAILINGS} = \text{REST\_B} \times \text{DENSITY\_B} + (1 - \text{REST\_B}) \times \text{DENSITY\_A}
\]

Having the parameters from borehole samples our further step for constructing minirallurgical and economic models should consist of using an appropriate allocation, extrapolation, interpolation or conditional simulation technique to model deposit for the strategic mine planning.

Examples

Some practical results of using ore-dressing predictions for constructing an economic block model of Kvodor multi-element deposit were published earlier (Whittle and Vassiliev, 1997). There we have utilized information about mean grain sizes of magnetite, apatite and baddeleyite minerals in different rock types under correction from test works to estimate expected mineral-dressing recoveries in blocks. To get optimized limits of the Kvodor open pit with Lerchs-Grossmann algorithm the Whittle Three-D package was used in connection with the Geolmark general mining program. The size of final regular block model was 1500x1500x900 meters with block dimensions of 30x30x15 meters. We have got rather different outlines for mining phases on every bench of the block model comparatively to ones without ore-dressing predictions as in the case of constant recoveries. The estimation results could be utilized in further designing and long-term planning for the open pit after co-ordination with design developed organization.

Another project is pushing ahead on Stoienskijy open pit mine to find the best strategy of mining for long-term planning. This open pit mine is the third largest iron ore producer in Russia. Due to geologic complexity of the deposit and its big size there was a decision to create more detailed
minerallurgical model of the site to investigate possibilities of optimizing open pit limits with more powerful Whittle products such as Four-D and Four-X. Low-grade quartzites of the deposit contain magnetite and hematite as main iron minerals but they have very strong spatial variations of grades comparatively with the nearby located Lebedinskiy iron ore deposit. Fifteen different rock types actually can be taken into consideration at the Stoilenskyi deposit. They include ten ore types inside orebodies for scheduling purposes. For bulk of samples without stereological data analysis it was possible to assign mean chords for magnetite and gangue in relation with according rock types.

In ore-dressing plant for size reduction/liberation of magnetite are placed ball mills sections and adjusted magnetic separation scheme. Both magnetite and hematite often occur in the same SMU and sometime it is difficult to make right the choice.

The deposit is modeling now with the help of Geoblock software with relational database tables. The software implements essential set of drillhole processing procedures and interpolation by ordinary kriging. Optimal pits were generated using Whittle optimizers with different recoveries in blocks after predictions. The results were compared with the pits generated without ore-dressing predictions, which indicate that the proposed method for individual magnetite reserve estimation gives an improvement comparatively to those based on common iron grades associated not only with magnetite but also with hematite and other iron contained nonmagnetic minerals. The optimal pits generated from the recoverable resource model were found and outlined smaller, sometimes for several work benches, than those generated with usual way. Because this can lead to changing of further mining strategy with respect to maximization of NPV, it is necessary now to make revision of previous projects and conclude new agreement with the project maintained institution.

Conclusions
The results of this paper’s investigations can be summarized as follows:

- It was suggested that mineral liberation spectra from borehole cores, SMU or entire ore body can characterize the potential minerallurgical ability of ores with regard to any individual phase of interest or grouped target minerals.
- The new approach in simulation of mineral liberation phenomenon and ore texture quantification has been developed. In a software program there were implemented procedures based on stochastic geometry and Markov chain analysis to predict the particle size-grade distribution in multi-mineral heterogeneous systems. This can help to forecast grades and yields of final concentrates, as well as the recoveries of target mineral/metal before crushing, grinding and separation circuits.
- The technique should provide the reliability and precision of modeling the size-grade distribution and ore-dressing prediction for any cut-off grades in SMU with different sizes.
- The interface to Whittle Three-D and Four-D pit optimization packages was implemented in new version of software to produce optimal profitable mining sequences, which can be useful for both designing and scheduling purposes.
- The approach has been used in iron ore body modeling, simulating ore-dressing parameters inside selective mining units, calculating recoverable reserves and preliminary optimizing open pit limits for iron deposits in Kolskyi Peninsular and Khursk Magnetic Anomaly region in Central Russia.
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References


