Where Four-D Continues on...

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Abstract

From a theoretical point of view Whittle Four-D optimisation marks a significant step in the development of knowledge in the field of pit design, namely: the development of a method by Lerchs and Grossmann and its computerisation for determining the optimal pit outlines. As a methodology Four-D also includes a solution to mine planning.

The present article gives an answer to the question of the future development of the methodology of Four-D optimisation in the area of mining operations planning, which hides big possibilities for the increase of economic effectiveness of mining ventures. For that purpose, the fundamentals of a new approach to planning are analysed.

An example is provided based on data related to an open pit mine for gold mining. Conclusions are drawn regarding the possibility of the new approach to planning giving a significant increase in the Net Present Value of pit projects.

Introduction

The development of a computer system as a matter of principle always creates progress in the scientific knowledge of a specific research field. An analogous approach was used in the development of Four-D for the needs of open pit mine design. The system provides a computer program to handle the two basic stages of design, namely: determination of the pit outlines and then planning of the mining operations within the pit outlines. Furthermore, with its original solution for generating a series of pit shells in accordance with a set of metal prices, Four-D provides a link between these two stages. The direction of this link reveals the fundamentals of a concept that the pit outline design and mine planning are only two phases of the solution of the common problem for finding the best NPV of the pit project.

If the method of Lerchs & Grossmann (1965) used in Four-D for determining the open pit mine outlines holds a place in contemporary surface mining theory and practice, such a qualification regarding the computerised approach of planning would be accepted as disputable. This approach is proposed, because the analysis of its principles of planning, and the practical results, show the possibility of reaching better economic results. This is confirmed in a recent investigation of R. Wooller (1995) who concludes that "it (Four-D) was not a complete open pit mine planning package...". His idea to show "where Four-D ends", however, can be considered only in the context of the computational procedures of the approach adopted to pit design and planning, but not in a purely theoretical aspect. In this respect the methodology of Four-D leaves doors wide open for discussion and further development of its content, with the clear understanding of its authors that this is the only way of maintaining its vitality.

Analysis of the Four-D methodology

The essence and advantages of the Four-D methodology are discussed in detail by its authors (Whittle, 1993; Tulp, 1994; Hanson, 1994; Wharton, 1996) and followers (Charbonneau, 1991; Barnes, 1995; Bertingshaw et al., 1995; Wooller, 1995; Prenn, 1995). That is why there is serious interest in an analysis of the future progress of this methodology which can outline the problems to be solved. In our opinion such problems are as follows:

1. *Determination of the optimum number of cut-backs which represent a grouping of pit shells into separate independent pit units, with each having its own working zone and mining front.*

The solution of the problem suggested in the Four-D methodology is based on the analysis of the graph of NPV versus pit shell number or the so-called pit-by-pit analysis (Wharton, 1996) which is too subjective, because it is not based on a scientifically grounded optimisation procedure. Obtaining the correct solution to the problem is significant because it is directly related to the macro-economy of the pit design. This solution can also provide the best outline for the initial open pit in accordance with the geological, geomechanical, technological and economic conditions, in the context of the strategy adopted for long term planning of the mining operations.

2. *Determination of the rational direction for the evolution of mining operations within the pit space.*

The direction of the evolution of the mining operations within the pit space is determined by the spatial location of the initial and drop cut on every pit horizon. It influences the waste quantity and its distribution in time for the whole life of the mine, the mining recovery and dilution, and qualitative characteristics of the ore exploited, as well as on the size of the NPV of the mining venture. That is why determining the rational direction of evolution of mining operations is one of the basic problems of pit design.

In contemporary research this problem is being treated mainly on the principle of high grading (Whittle et al., 1991) as a principle of planning and with the use of the division of the geological medium into blocks (Tolwinski, 1992; Wooller, 1992; Huang, 1993; Onur et al., 1993). This approach can not give a complete solution to the problem because it ignores the basic principle of long term planning which is the principle of waste deferment (Halatchev et al., 1994).

In the present Four-D methodology this problem is not formulated.

3. *Development of an optimisation approach for pit production scheduling on the basis of a well-established mathematical optimisation procedure.*

The FDAN analysis does not use a mathematical optimisation tool like linear, non-linear and dynamic programming, etc. The use of such a tool is required for the solution of pit production scheduling because the formulation of the problem for finding the maximum NPV of the project, having regard to the economic conditions and technological constraints of the mining venture, has an optimisation character. FDAN analysis fixes only the bounds (worst NPV-best NPV case) of the feasible domain for generating production schedules, of which there are an unlimited number. In addition the bound of the best NPV case has a purely theoretical meaning because it does not reflect the real conditions of mining operations (Halatchev, 1996).

FDAN analysis also has the following shortcomings:

- the pit shells which are generated do not obey the technological requirements of planning, such as ensuring minimum working widths on the working benches for dispatching the basic mining equipment (shovels, trucks), as well as the location of normal initial and drop cuts.
- the use of cut-backs with working widths bigger than the minimum ones from a technological point of view always leads to the increase of the quantity of waste removal and decrease of NPV. Hence, the working widths of the working benches are a limiting factor of the effective exploitation of a given open-pit mine and they must be optimised.
- the pit production scheduling on the principle of the block presentation of the ore deposit is too sophisticated. It neglects the main technological requirement for leading the mining operations on bench panels which can have a variable width. The best evidence of this stand is the real mining practice where the shovels 'like' to work on panels but not on 'inconvenient' blocks.
- the effective use of the available shovels as a specific problem of mine planning is not considered in the FDAN possibilities. This problem deals with the optimisation of shovel capacity in the context of optimisation of the pit mining rate.
- the stochastic nature of shovel production in real mine planning is not taken into account. From a mathematical point of view, this production represents a random quantity and it always varies within some admissible (min/max) range which depends on geomechanical, technological and technical factors. This circumstance predetermines the stochastic nature of the pit production regarding the admissible mining rate.

4. *Incorporation of risk analysis for a quantitative assessment of the degree of uncertainty in the economic and technological parameters of planning information which is used.*

The sensitivity analysis method computerised in Four-D is a mathematical procedure having a low level of a logical content. It only suggests the possibility of evaluation of the degree of the influence of the variables which affect NPV, and a consequent acceptance of decisions for an effective management of those factors which have a large influence. All comments made in recent investigations on sensitivity analysis as a mathematical tool are exactly on these lines (Roditis, 1993; Hanson, 1995; Wharton, 1996; Halatchev, 1996a).

The main disadvantage of sensitivity analysis is that it can not assess quantitatively the degree of uncertainty in the information used in mine planning, which is a priority of risk analysis. Namely this fact gives grounds to conclude that risk analysis takes sensitivity analysis a step further because it is also better grounded mathematically. Here it is worth noting that there is not currently a universally recognised risk model of mine planning from a theoretical point of view. The variety of risk models developed at present is subordinated to different strategies and targets of the mining venture, which makes the problem of developing a universal risk model exceptionally complex.

The solution of the above-formulated problems is a serious research challenge and it, undoubtedly, will allow the updating of the Four-D methodology.

New methodology of planning

A new methodology of pit production planning is suggested which is a consequence of the Four-D methodology for staged exploitation of ore deposits. It treats two mutually associated problems: the optimisation of the number of cut-backs of the pit complex and the optimisation of the long term production schedule.

The technology of Four-D for generating pit shells is used as a point of departure of the solution of the problem for optimising the number of cut-backs. This technology has a purely engineering sense and represents a specific break-up of the geological medium which, as a principle of solution, is used widely in contemporary engineering sciences. A good example in this relation is the application of numerical methods such as Finite Difference Method, Finite Element Method and Boundary Integral Equation Method which also use a break-up of the geological medium for the solution of problems in the fields of Rock and Soil Mechanics, Hydrogeology, Engineering Geology, etc.

The solution of the first problem is obtained on the basis of a method developed for optimising the production schedule of a pit complex (Halatchev, 1993; Halatchev et al., 1994; Halatchev, 1996b). The optimal variant of the pit shell grouping into cut-backs as independent pit units is determined with a comparative analysis of the assessments of NPV of the long term planning obtained for all the theoretically possible variants of pit shell grouping into pit units, and with the use of constant values of the economic variables of the NPV function (Halatchev, 1995; Halatchev, 1997).

The solution of the second problem is reached with the development of the above-mentioned method for optimising the production schedule of a pit complex. It expresses the idea of forming a feasible optimisation domain based on a cumulative representation of the waste and ore quantities, which lower and upper bounds, correspond to the extreme (best NPV- worst NPV) cases of the evolution of the mining operations. The feasible domain allows the determination of the optimum pit production schedule in accordance with the economic conditions and technological and geomechanical constraints of open pit mine exploitation.

The results of Four-D also allow the building of a feasible optimisation domain. The real lower bound of the domain, however, must be built with the arrangement of the mining operations on the working benches with minimum working widths, ie. using the principle of bench increment planning. There is a possibility for using the principle of pit shell increment planning if we can guarantee minimum working widths on the working benches. This requirement is explained by the fact that the working benches, with normal working widths determined in accordance with the requirements of the basic mining equipment, represent purely technological elements while the pit shells generated are purely economic elements, because their size is a function only of the metal price. In this connection the practical application of the principle of the bench increment planning always gives the best economic results for long term mine planning, in contrast to the pit shell increment planning. Here it is worth stressing that the pit shells generated by Four-D suggest exceptionally useful information regarding the direction of the mining front movement, and for the choice of the logical mining system. It is preferable to use this information in the arrangement of mining operations on the working benches.

The main specific feature of the method developed for optimising the production schedule of a pit complex is the fact that it incorporates the two basic principles of mine planning, which are waste deferment and high grading in competition with each other. The first principle makes a difference between the waste and ore, while the second principle makes a difference between the quality of separate parts of the ore body. It can be stated that the principle of waste deferment is the main principle of long term planning, because it deals with the macro-economy of the planning. As a consequence of its practical application, a great deal of capital investment can be used for other purposes rather than being kept idle in the mining production system.

The basic advantages of the method are as follows:

- it is intended for long- and short-term planning of mining operations in open pit mines.
- it is based on the 'best NPV-worst NPV' concept of a cumulative presentation of the waste and ore quantities of an open pit mine or individual pit units of a pit complex, which makes it universal and applicable to the exploitation of any type of deposit - gold, copper, iron, coal, sand.
- it uses a mixed linear-integer programming optimisation technique as a basis for optimisation models of pit production scheduling.
- it uses the bench panel as a real technological element of mine planning in contrast with the bench block of the ore block model, which is not sufficiently relevant to the problem of the planning of mining operations.
- it allows the optimisation of the production schedule of a single open pit mine or a pit complex represented by individual pit units as cut-backs or open pit mines involved in a common production process.
- it is theoretically developed and can be applied at the exploitation of a hypothetical ore deposit presented simultaneously by "n" number of basic types of ore and "m" number of secondary types of ore (the definitions of these ore types are given in Halatchev, 1993).
- it allows fulfilment of the technological requirement for a stabilisation of the mining rate of the open pit mine or pit complex in order to maximise effective utilisation of the available shovels. It is done with the optimisation of the shovel capacity against time, which takes into account the economic conditions of the utilisation of the available shovels and their maximum possible production capacity. The method also suggests a variant that uses integer variables so that an integral number of the shovels is used.
- optimisation of the pit production schedule results in an optimisation of the geometry of the working slopes within the working zone of the open pit mine or pit complex, which is the main target of long term planning. It means that this geometry can change like opening or closing of the pit cone within the working zone or zones in accordance with the economic conditions, which change with time. This type of optimisation gives the best economic results as well as the possibility of a flexible reaction to variations in market conditions.
- it is universally applicable regarding the mining technologies used.
- it is simple and easy to apply.

Case study

Mining object of investigation: The present investigation is based on the use of Four-D tutorial output data files (fdtut.par, tut1.res), which are for a small gold mine. Forty pit shells are generated for the chosen range of MCOSTM values. The total ore reserve within the final pit outline is 7,308,240t and is spread over 23 benches with a height of 10m. The gold ore is divided into two types: basic type of ore (BTO) and secondary type of ore known in the mining practice as marginal-subgrade material (MSG), using the cut-off grade of 1g/t. The basic type of ore is processed in a processing plant while the marginal-subgrade material goes to a stockpile for heap leaching. The annual pit production of the BTO are fixed initially as constant quantities (Table 1, column 6) and their cumulative representation forms the Annual Ore Production Function (AOPF).

Computer implementation: The computer code FDLT was used for the case study (Halatchev, 1995). The code is written in QuickBASIC - Version 4.5 of the Microsoft Corporation and runs on IBM PC compatible computers.

Input data: The following input data are used:

life of mine - 6.14 years

interest rate - 10%

unit sales cost - \$1.00/oz

unit operating cost of waste removal - \$3.22/t

unit operating cost of ore mining - \$1.94/t

unit operating cost of basic ore processing - \$7.37/t

unit operating cost of MSG processing - \$4.55/t

time costs - \$980,000/year

Two models of shovels are used, one with an annual average production of 1.5 Mt (2 shovels) and the other with an annual average production of 2.5 Mt (1 shovel). The shovels can give a possible mining rate as follows: minimum - 3.850 Mt/year, average - 5.500 Mt/year and maximum - 7.150 Mt/year. The assessments of the possible mining rate are determined on the basis of the 3-Sigma statistical rule toward the standard deviations (150,000 t/year and 250,000 t/year respectively) of each shovel model production.

unit purchase cost of the production capacity of the 1st shovel model - \$6.00/t

unit purchase cost of the production capacity of the 2nd shovel model - \$4.00/t

penalty from the decrease of the production capacity of the 1st shovel model - \$2.00/t

penalty from the decrease of the production capacity of the 2nd shovel model - \$1.00/t.

Results: Two scenarios of the pit design and planning are envisaged as follows:

- **Scenario A:** a single open pit mine presented by all generated pit shells.

The cumulative graph of the open pit mine is illustrated in Figure 1. The curve for the worst NPV case reflects the waste and BTO & MSG quantities related to the whole number of benches (23). The curve for the best NPV case is built with the waste and BTO & MSG quantities related to the whole number of the pit shells (40). The curve of the optimum solution is located within the feasible domain of the graph. The discounted cash flows having a relation to these 3 curves are given in Table 1 (columns 2,3,4), which also summarises the results on the optimum pit production in the waste, BTO and MSG as well as the relevant quantities of BTO metal and MSG metal. The assessments of the NPV of the extreme and optimum cases are given in Table 5 (columns 2,3,4). The NPV for the optimum solution is \$76,391,888.

The results on the formation of the mining rate of the open pit mine are given in Table 3 and they allow it to be concluded that the optimum mining rate is a function of the optimum production of each shovel model, which is determined in accordance with the assessments used for the unit purchase cost of the production capacity and the penalty of its decrease. The results also show when new capacity is to be added and the capacity decrease of each shovel model in time, that is the specific feature of the optimisation model of the pit production scheduling.

The factors of compromise as a criterion for the evaluation of the effectiveness of the optimum solution for the pit production schedule (Halatchev, 1996b) are given in Table 5. This criterion treats two aspects of mine planning - technological and economic, which are assessed with the technological factor of compromise ($\mathbf{F}_t^c$) and the economic factor of compromise ($\mathbf{F}_e^c$) respectively. The common factor of compromise ($\mathbf{F}_c$) for the optimum solution represents a multiplication of the both factors and its assessment is 5.48%.

- **Scenario B:** an open pit complex presented by 3 stages: an initial pit consisting of pit shells 1 to 23; cut-back No. 1 consisting of pit shells 24 to 28; cut-back No. 3 consisting of pit shells 29 to 40.

The initial representation of the cumulative graphs for the open pit complex is illustrated in Figure 2. The graphs are transformed into a common cumulative graph of the open pit complex with the use of a technique developed (Fig. 3). The feasible domain of the re-arranged graph allows the determination of the optimum solution of the production schedule because it meets the requirement for insuring enough quantity of the BTO in accordance with the fixed AOPF. The curve of the optimum solution is located in the common feasible domain and follows the curve of the best NPV case very closely. The NPV of the optimum solution is \$78,889,632, and the relevant factor of compromise is 1.37% (Table 5). The detailed distribution of the discounted cash flows of the optimum and extreme schedules is presented in Table 2, which also summarises the results on the optimum quantities of the waste, BTO and MSG for the used time discretion as well as the optimum quantities of the metal for the both ore types.

The results of the optimum mining rate of the open pit complex are given in Table 4.

Figure 4 gives a notion about the common upper bound functions regarding the waste, BTO and MSG from the arrangement of the mining operations for the worst NPV case versus the time of exploitation of the open pit complex. Its analysis shows that the Common Upper Bound Basic Ore Function (CUBBOF) is located entirely over the AOPF which means that this case of the mining operations arrangement ensures enough quantity of the BTO for meeting the planned annual quantities for the pressing plant.

The time of the transition commencement of the mining operations from the first pit to cut-back No. 1 is T_1 = 2.70 years from the beginning of the exploitation of the open pit complex, while the time of the transition commencement to cut-back No. 2 is T_2 =5.59 years. These times are related to the optimum solution of the production schedule.

The requirement on the dynamics of mining operation evolution of both transitions is fulfilled in accordance with the theoretical treatments of the developed methodology of planning (Halatchev, 1993). It deals with the guarantee of normal technological conditions for leading surface mining operations at the transition from one stage to another.

The comparative analysis of the results on the NPV and factor of compromise for both scenarios shows that the scenario for the exploitation of the gold deposit with an open pit complex is considerably more effective than the scenario for using an open pit mine. In spite of the fact that both the ore reserve and values of the economic variables in the NPV function are constants in the present investigation, the NPV of the optimum production schedule for the open pit complex is higher than the NPV of the optimum production schedule for the open pit mine. Graphically it is illustrated with the metamorphosis of Figure 1 into Figure 3, which leads to narrowing the feasible domain of the common cumulative graph of the open pit complex and in this way the curve of the optimum solution comes nearer to the curve for the best NPV case. Namely this is the explanation of the mechanism for the high economic effectiveness of the strategy for staged exploitation of ore deposits that puts a strong emphases on the principle of waste deferment at the long term mine planning.

The results from the example provided are also a proof of the advantage of the strategy for staged exploitation as the most advanced in pit design theory and practice at present which is adopted as a basis in the Four-D methodology.

Conclusions

1. The development of the Four-D methodology is a process that is a consequence from the philosophy of the progress of scientific knowledge as a whole and, in private, in the field of the pit design and mine planning.
2. The new methodology suggested is an exceptionally good alternative for an effective long-term and short-term planning of mining operations. It is universal for practical purposes regarding the mining technologies used and the types of deposits exploited, as well as suggesting a mathematically based optimisation procedure for scheduling the pit production with a high degree of adequacy of the modelling of this technological-economic problem.
3. The results obtained from the solution of the example provided are a serious proof of the possibilities of the new methodology for a huge increase of the profit from mining ventures.

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The author also shares the opinion that the international scientific forums and collaboration are the key to the successful solution of the global problems of surface mining.

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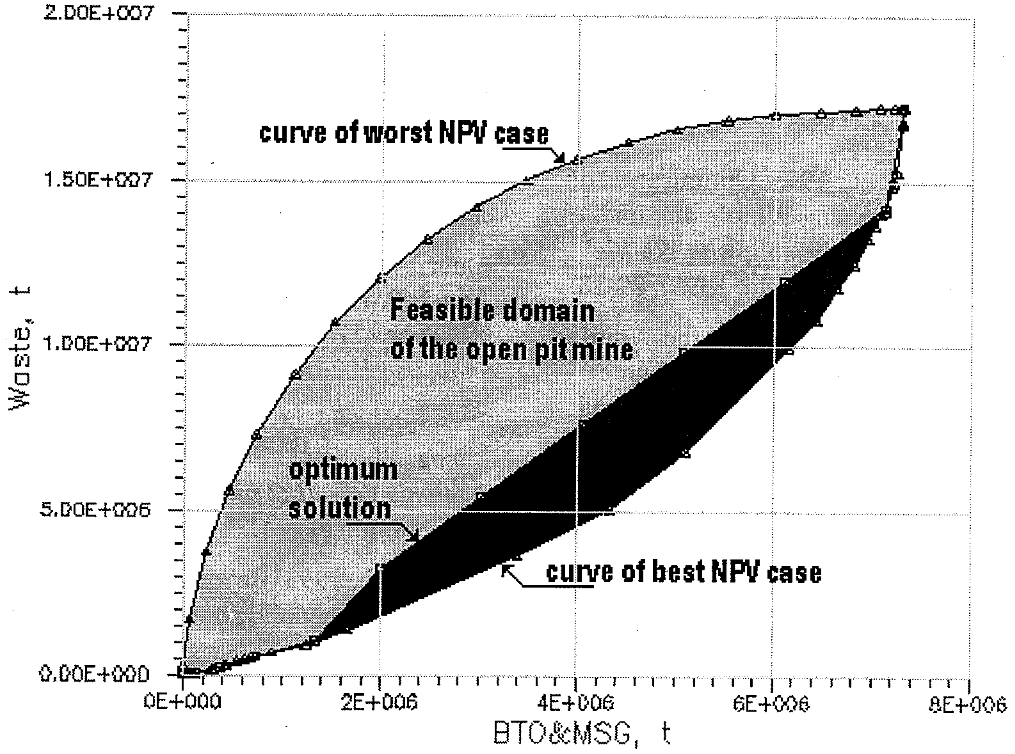

Fig 1. Cumulative graph for the open pit mine with final outlines.

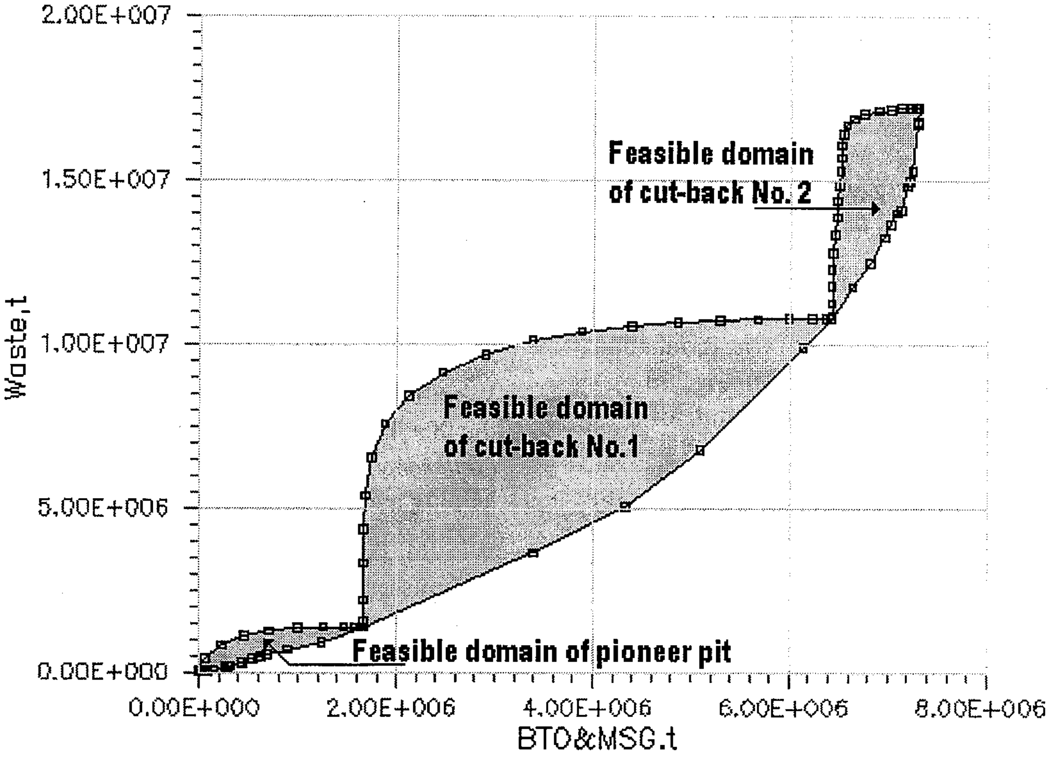

Fig 2. Initial representation of the cumulative graphs for the open pit complex.

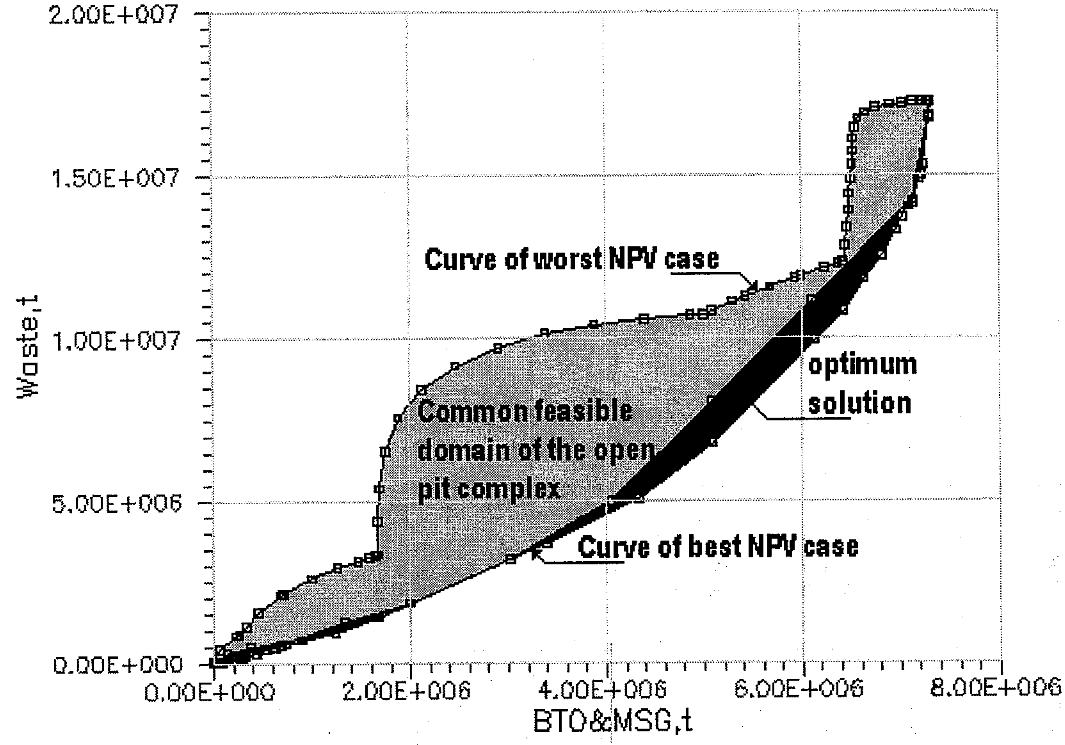

Fig 3. Re-arranged common cumulative graph for the open pit complex.

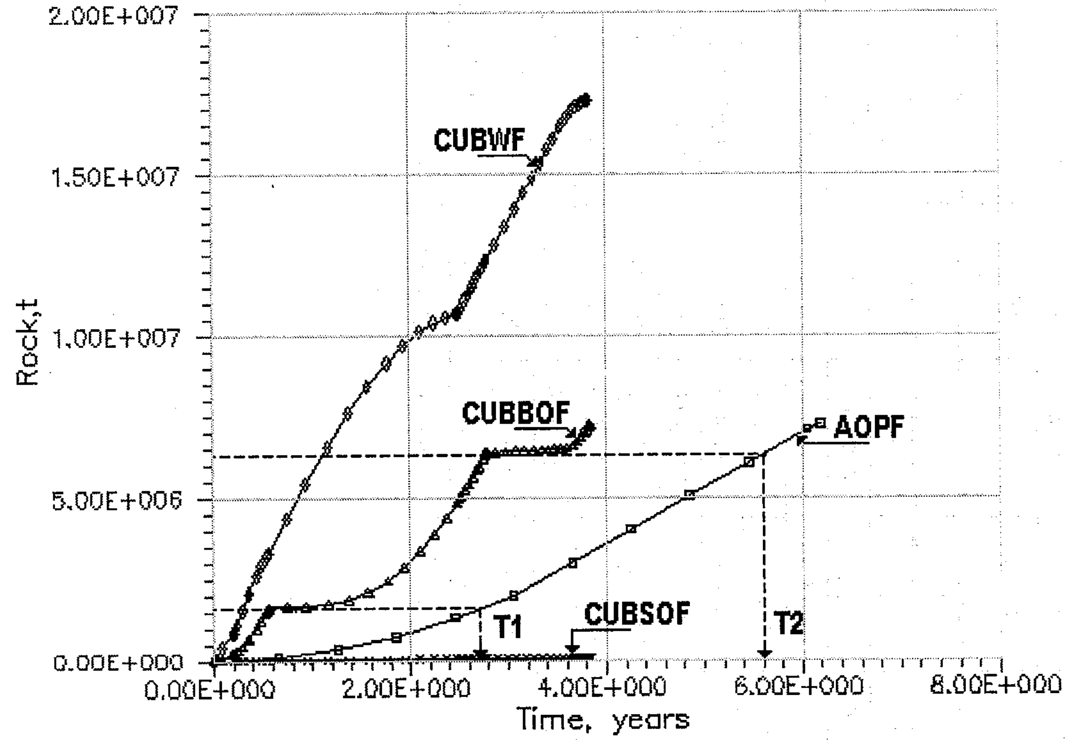

Fig 4. Common Upper Bound Functions for the open pit complex - worst NPV case.

Table 1. Basic results of the optimisation of the production scheduling of the open pit mine- ***Scenario A***.

Time, years	**Discounted** best case	**cash** worst case	**flows, US$** optimum case	**Waste,** t	**BTO*,** t	**MSG,** t	**BTO metal,** g	**MSG metal,** g
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
0.0	0	0	0	0	0	0	0	0
0.6	4873207	-6078394	1882337	60681	130000	0	19577	0
1.2	10757037	-8614730	5714469	181734	240000	0	27704	0
1.8	17328812	-8534357	11414682	327773	360000	1387	34836	39
2.4	27601404	-4648463	20987264	452082	590000	11350	54648	320
3.0	37610680	1347205	26723672	2238330	650000	9800	60583	255
3.6	52928852	13357150	38693192	2191830	1020000	8334	95807	232
4.2	65479020	26398004	50001748	2190740	1020000	9413	89532	215
4.8	74667400	39923820	60673456	2192940	1020000	7218	83308	230
5.4	84753064	53560616	70761640	2190240	1020000	9918	95602	272
6.0	84425072	66925332	80273288	2194940	1020000	5220	66725	139
6.14	79418752	68583256	76391888	3044560	162640	12960	8649	332

(*)Note: the basic type ore quantities are fixed initially as AOPF.

Table 2. Basic results of the optimisation of the production scheduling of the open pit complex - ***Scenario B***.

Time, years	Discounted	cash	flows, US$	Waste, t	BTO*, t	MSG, t	BTO metal, g	MSG metal, g
	best case	worst case	optimum case					
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
0.0	0	-108192	0	0	0	0	0	0
0.6	4873207	2029188	3558408	303859	130000	0	18300	0
1.2	10757037	7316059	9871753	193859	240000	0	28801	0
1.8	17328812	12134209	17210588	72471	360000	1388	35016	39
2.4	27601404	19974842	26778126	649319	590000	11481	54226	325
3.0	37610680	20055562	37457588	592245	650000	8555	61004	250
3.6	52928852	33163234	52771660	1337286	1020000	9833	95897	233
4.2	65479020	48658180	64407520	1849942	1020000	8144	89103	222
4.8	74667400	62597564	71748640	3073390	1020000	8355	84286	230
5.4	84753064	75599696	81327512	3072314	1020000	9454	95946	261
6.0	84425072	74195072	83896216	3076354	1020000	5431	65743	141
6.14	79418752	74906984	78889632	3044802	162640	12959	8649	332

Table 3. Results on the formation of the mining rate - *Scenario A*.

Time, years	**Shovel** new capacity, t	**model:** capacity decrease, t	**1-st** production, t	**Shovel** new capacity, t	**model:** capacity decrease, t	**2-nd** production, t	**Mining rate,** t
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
0	0	0	0	0	0	0	0
0.6	0	0	0	190681	0	190681	190681
1.2	0	0	0	231053	0	421734	421734
1.8	0	0	0	267427	0	689161	689161
2.4	0	0	0	364271	0	1053430	1053430
3.0	948135	0	948135	896568	0	1950000	2898135
3.6	322025	0	1270160	0	0	1950000	3220160
4.2	0	0	1270160	0	0	1950000	3220160
4.8	0	0	1270160	0	0	1950000	3220160
5.4	0	0	1270160	0	0	1950000	3220160
6.0	0	0	1270160	0	0	1950000	3220160
6.14	0	0	1270160	0	0	1950000	3220160

Table 4. Results on the formation of the mining rate - ***Scenario B***.

Time, years	**Shovel** new capacity, t	**model:** capacity decrease, t	**1-st** production, t	**Shovel** new capacity, t	**model:** capacity decrease, t	**2-nd** production, t	**Mining rate,** t
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
0	0	0	0	0	0	0	0
0.6	0	0	0	433859	0	433859	433859
1.2	0	0	0	0	0	433859	433859
1.8	0	0	0	0	0	433859	433859
2.4	0	0	0	816941	0	1250800	1250800
3.0	0	0	0	0	0	1250800	1250800
3.6	1116323	0	1116323	0	0	1250800	2367123
4.2	0	0	1116323	510994	0	1761794	2878117
4.8	1223676	0	2339999	0	0	1761794	4101793
5.4	0	0	2339999	0	0	1761794	4101793
6.0	0	0	2339999	0	0	1761794	4101793
6.14	0	881356	1458643	0	0	1761794	3220437

Table 5. Results for the Factors of compromise.

Mine	$\mathbf{NPV}^{min}$, US$	$\mathbf{NPV}^{max}$, US$	$\mathbf{NPV}^{opt}$, US$	$\mathbf{F}_c^t$, %	$\mathbf{F}_c^e$, %	$\mathbf{F}_c$, %
1	**2**	**3**	**4**	**5**	**6**	**7**
Single open pit	68583256	79418752	76391888	19.60	27.93	5.48
Open pit complex	74906984	79418752	78889632	11.69	11.73	1.37

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