RECOVERABLE RESOURCE MODELS AND OPTIMIZATION

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ABSTRACT

The purpose of this paper was to try to provide a practical appreciation of the logic, the benefits and the problems associated with the estimation of reserves using recoverable resource modelling methods.

The uncertainty associated with the estimation of small block grades in deposits which show extreme grade variation and complex geometry should not be ignored. It leads to serious errors in resource estimates at a local and global scale.

Such errors must impact on pit design and scheduling no matter what optimization method is used.

Recoverable resource estimation models provide a means to overcome the impact of some of these errors in local and global estimation. Their use results in rather coarse panel models which are less desirable for design and scheduling purposes but which provide an implicit description of the local uncertainty in the knowing the grade distribution. Some initial evidence suggests that models which incorporate this uncertainty provide a better basis for pit optimization.

INTRODUCTION

This paper is concerned with the effect of grade estimation error on the optimization of open pit reserves using linear or dynamic optimization methods.

Presently, many different methods are used to estimate the grades of mining blocks within resource models which are used as input to optimization programs. The choice of block size is commonly driven by engineering requirements like the need for accurate modelling of pit slope angles. The choice of method used to estimate block grades is often based more on the prior experience of those managing the construction of the resource model than on a clear understanding of strengths and weaknesses of the various methods (Schofield 1990).

Some general comments that apply to the construction of any block resource estimation model are;

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- Different estimation methods generate different estimation errors in estimating the grades of blocks.
- Geostatistical estimation methods provide ways to describe the nature of the estimation error and some methods are more effective than others in achieving this description.
- In general, the size of the block estimation error is inversely related to the size of the block.
- In general, the size of the estimation error increases with the magnitude of the estimate.

Uncontrolled block estimation errors can have very serious effects on resource estimates and the pit optimization and scheduling regardless of the optimization method used;

- global resources will be badly estimated
- pits will be poorly designed
- scheduling may be completely meaningless

In the 1970's and 80's, several geostatistical methods were developed to provide a workable solution to the problem of estimating recoverable resources, particularly for precious metal and other deposits where the grade variation and geologic complexity can be extreme. Lognormal kriging and other methods with the unlikely names of Indicator Kriging, Probability Kriging, Disjunctive Kriging, Multigaussian Kriging all provide a means of estimating more precisely the resources in a deposit.

These methods are much more difficult to understand, implement and use than any of the more traditional methods and the solution they provide is less digestable to those concerned with generating well engineered and visually appealing pits.

There is growing evidence to indicate that for deposits with extreme grade variation which includes most gold deposits and a large number of base and multi-metal deposits, the use of estimation methods which do not provide some means to describe the estimation error in grade can lead to costly mistakes in resource estimation and mine design.

THE NATURE OF THE ESTIMATION ERROR

No matter what estimation method is used to estimate the grade of a mining block, there will almost always be a difference between the estimate and the actual grade of the block.

Most of the commonly used estimation methods like sectional-polygonal methods, the inverse distance methods and geostatistical methods generate estimates for which the average estimation error is close to zero ie. approximately globally unbiased. This is achieved by forcing the sum of the weights which are applied to samples used for each estimate to be unity. With the use of these methods, one has no control over the estimation error that might be generated for any specific block but the mean of the estimation errors generated for all blocks estimates can be maintained close to zero.

$$\frac{1}{N}\sum_{i=1}^{N}\{g^*(x_i)-g(x_i)\}\approx 0.\text{ for } N \text{ block estimates}$$

where g^* represents the estimate of a block grade and g represents the actual block grade.

The use of the ordinary kriging method allows some additional control over the statistics of the estimation errors which the traditional methods do not. The ordinary kriging sample weights are chosen in such a way as to minimize the variance of the estimation errors.

$$\text{minimize } \sigma_R^2 = Var\{g^*(x)-g(x)\}$$

The expression of the ordinary kriging estimation variance in terms of the semi-variogram $\gamma(h)$ is

$$\sigma_R^2 = \sum_{i=1}^{n} w_i \cdot \gamma_{i0} + \mu$$

where w_i are the kriging weights and μ is the lagrange parameter.

This expression does not provide a useful measure of the local accuracy of an individual block kriging estimate because it does not take into account the uncertainty related to the magnitude of grade sometimes known as the proportional effect (*Isaaks and Srivastava, 1989*). The expression does not incorporate directly or indirectly the sample grades in the local neighbourhood.

The use of an ordinary kriging estimator with widely spaced sampling to estimate the grades of small blocks that reflect the anticipated selection strategy to be used in mining has two major drawbacks;

- <u>SMOOTHING OF ESTIMATES</u>

Smoothing is an inevitable consequence of estimation which involves weighted averaging of the sampling data and its effects may be appreciated in a number of ways.

The smaller the block size compared to the sample spacing, the greater is the overall smoothing. Blocks which are further away from any sampling will have greater smoothing in their grade estimates compared to those close to sample locations.

The histogram of estimated block grades will have a number of different characteristics to the histogram of actual block grades; it will have a smaller variance, will be more symmetric and will have fewer high and low grades.

Applying an economic cutoff grade to the histogram of estimated grades of small blocks cannot provide an accurate estimate of recoverable resources or reserves.

- <u>PROPORTIONAL EFFECT</u>

This effect is one of the most important sources of error in local estimation of block grades and is not taken into account in the ordinary kriging variance. The uncertainty of knowing the grade of a block usually increases with the estimated grade of the block. Consequently, there is often a much greater uncertainty attached to high grade block estimates. This fact limits the practical freedom to increase the cutoff grade in order to recover higher ore grades.

METHODS TO DESCRIBE LOCAL ESTIMATION ERROR

During the 1970's and 80's, the geostatistical community expended a great effort to develop methods to provide a better description of the uncertainty associated with knowing the grade of a particular block in a model. The needs of a number of industries including the mining industry provided some of the motivation for this effort.

In describing the uncertainty attached to the grade of particular block in a model, essentially one is concerned with describing a histogram or a cumulative histogram of possible grades for that block. One would expect that histogram to be different for each block because the configuration of samples and the sample values informing each block are usually different. Figure 1 provides some graphic illustration of these concepts. Two simple sample configurations around a block are shown together with an intuitive idea of the histogram of possible grades for the block. The configuration with less samples has a much larger range of sample grades and therefore admits a greater uncertainty for the block grade.

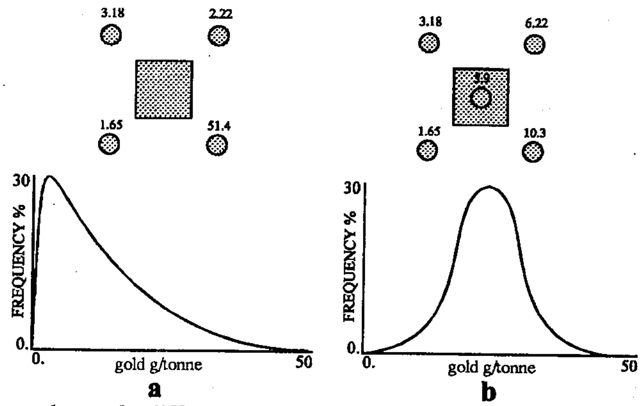

Figure 1 Examples of different sample configurations for block estimation. Configuration **a** has fewer samples with a large range of sample values. Configuration **b** has more samples with a much smaller range. Intuitively the histograms of possible grades for the blocks are judged to be quite different.

Estimation methods such as Indicator Kriging and Probability Kriging (*Sullivan, 1984*), Log-normal Kriging and Multigaussian Kriging (*Verly 1984*) and Disjunctive Kriging (*Jackson and Marechal 1989*) have been designed primarily to provide a useful description of histogram[1] of possible grades for a block given the local neighbourhood sampling data $f(g(x_0)|g(x_i), i = 1,....., n)$ where x_o denotes the location of the block being estimated with the n local data $g(x_i)$. It is not the purpose of this paper to discuss the details of these methods and their many variations which are currently available in various software packages. Some comments on their major advantages and disadvantages are;

- **<u>INDICATOR KRIGING AND PROBABILITY KRIGING</u>**

The major advantage of a strict Multiple Indicator Kriging (MIK) is that it allows an explicit and detailed description of the spatial continuity at different levels of grade to be used in the modelling. This may be important if mineralization results from the superimposition of several mineralizations with different geologic properties. The other estimation methods do not have this advantage or flexibility in modelling.

1. The histogram of a set of grades is a known non-linear function of grade denoted $f(g)$. The methods to estimate the local histogram are sometimes known as **non-linear** estimation methods.

The primary disadvantage of these methods is that they can occasionally produce inconsistent results even if properly implemented. In the same manner in which ordinary kriging can produce occasional negative estimates when properly implemented, Indicator Kriging and Probability Kriging can produce estimates of local conditional probability which are less than zero or greater than one. When a negative estimate arises using ordinary kriging, an optimal solution is to reset the estimate to zero. In most cases, such inconsistencies with the Indicator methods can be reconciled in a similar manner.

Another important disadvantage of these methods is that they are difficult to implement and use correctly. There are many different implementations in various software packages including variations of Median Indicator Kriging (*Lemmer 1984, Goovaerts 1994*) which have more specific conditions for application.

- **LOG-NORMAL, DISJUNCTIVE AND MULTIGAUSSIAN METHODS**

These methods are all based around the use of the Normal or Gaussian distribution (classical bell shaped distribution) in its bivariate or multivariate form and for this reason, they are generally known as Parametric Methods.

The major advantage of these methods is their comparative ease of use if they have been properly programmed. An important characteristic of the bivariate and more generally the multivariate Normal model is that they are completely described by a single variogram (*Verly 1984*).

There is no need for the multiple variogram calculation and modelling that is usually required with the Indicator methods.

However, these methods can be very difficult to understand for the lay person and have almost none of the intuitive appeal of the Indicator methods. They also lack flexibility for modelling complex mineralizations. The implicit properties of these Parametric Models may not be compatible with the properties of the mineralization under study (*Journel and Alabert 1989*).

BLOCK ESTIMATION ERROR

In open pits, unless there are clear visual geologic controls for ore waste definition which can serve as a mining ore boundary, ore selection is usually based on interpretations of grade control sampling and some minimum mining width or selection unit size.

Another consideration in the estimation of recoverable resources is that the histogram of grades initially provided by either the Indicator Methods or the Parametric Methods is based on sample support, not block support. Even if block kriging[2] is used (*Isaaks and Srivastava 1989*), the result represents the average histogram based on sample support within the block. It is not a histogram of possible grades for the block. For recoverable resource estimation, a histogram of grades based on block support is required. With the Log-normal and Multigaussian methods (Verly 1984), correct and consistent histograms of grade based on block support can be obtained. With the other methods, heuristic techniques can be used to provide useful results (Journel and Huijbregts 1978, Isaaks and Srivastava 1989).

2. In block kriging, the block is described by a set of regularly spaced points in the block.

Smoothing and Interpretation

The use of the Indicator or Parametric methods to estimate the histogram of possible blocks grades for a particular block in a model does not overcome the problem of smoothing associated with the estimation of small blocks. In the same way that estimation of the grades of small blocks from widely spaced data will tend to provide very smoothed estimates with similar grades (while the actual grades will vary significantly), these methods will provide histograms of possible block grades which are all very similar. How can such estimates of histograms of block grades be used to improve the estimates of recoverable resources?

A useful answer to this question lies in an interpretation of the average histogram of possible block grades over volumes which are sufficiently large to overcome the smoothing problem of estimation and also to ensure that the number of blocks within the volume is a reasonable statistical mass (at least 20). These large volumes are generally referred to as panels. The smaller mining units which comprise the panels are called smu's or blocks.

For many mineral deposits and particularly gold deposits, the minimum size of a panel to overcome smoothing of estimates is usually the average drill hole spacing which may be between 25 and 50 meters. The size of a mining block or minimum mining width is often around 5 meters or less.

The idea is to interpret the average histogram of block grades over the panel volume as the histogram of blocks grades within that volume estimated from grade control data. Applying a cutoff grade to the histogram will provide an estimate of the tonnage and grade of material that will be recovered from that panel at the time of mining.

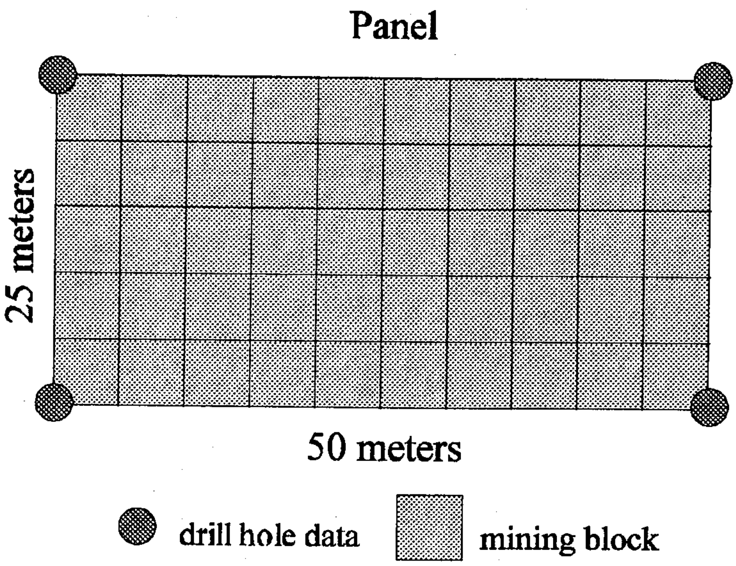

Figure 2: An example of a panel informed by four corner drill holes and comprised of 50 mining blocks.

It is important to appreciate that no clear understanding of the locations of those blocks that will be recovered as ore can be provided at the time of resource estimation. The high uncertainty associated with knowing whether an individual block in the panel is ore, has been spread over a larger number of blocks resulting in a better knowledge of the tonnes and grade of ore that can be recovered over all the blocks. In this sense, the panel size represents the limit of spatial definition of the grade distribution in the deposit. The notion of the panel and the histogram of blockgrades is illustrated in Figure 2.

IMPLICATIONS FOR MULTI-METAL DEPOSITS

Estimating recoverable resources for multi-metal deposit such as a copper-gold deposit where both metals make important contributions to the project viability requires an extension of the previous discussion concerning the histogram of block grades within a panel.

Estimating resources in a copper - gold deposit for example is often approached in either of two ways;

- **PRIOR APPLICATION OF EQUIVALENCE**

  The copper and gold sample grades are reduced to a single grade variable by the application of an equivalence formula to convert say gold grade into an equivalent copper grade. Resources of equivalent copper grade are estimated.

  The major drawback of this approach is that when the basis of equivalence (which is usually metal price) changes, the entire resource needs revision.

- **POST APPLICATION OF EQUIVALENCE**

  Separate block models of estimated copper and gold grade are constructed and the equivalence formula is applied to the block estimates. This avoids the problems of resource revision when metal prices change.

However, the problems related to smoothing of estimates in small blocks and proportional effect must still be addressed and to do this, the relationship between copper and gold must be more clearly understood.

Indicator Kriging can be used to estimate the histograms of block gold grades and block copper grades in a panel model but one is then faced with applying equivalent cutoff grades to two histograms and obtaining different results. This dilemma can be resolved through the use of the local bivariate histogram of copper and gold as shown in Figure 3. The equivalent copper-gold cutoff grade appears as a line on the plot with the equation.

$$eCu = Cu(\%) + r \times Au(g\ /\ t)$$

where r usually incorporates the ratio of gold price to copper price and other factors such as recoveries. The proportion of blocks to the right of this line represents a estimate of the proportion of the panel that will be recovered as ore with block selection based on the equivalent cutoff grade.

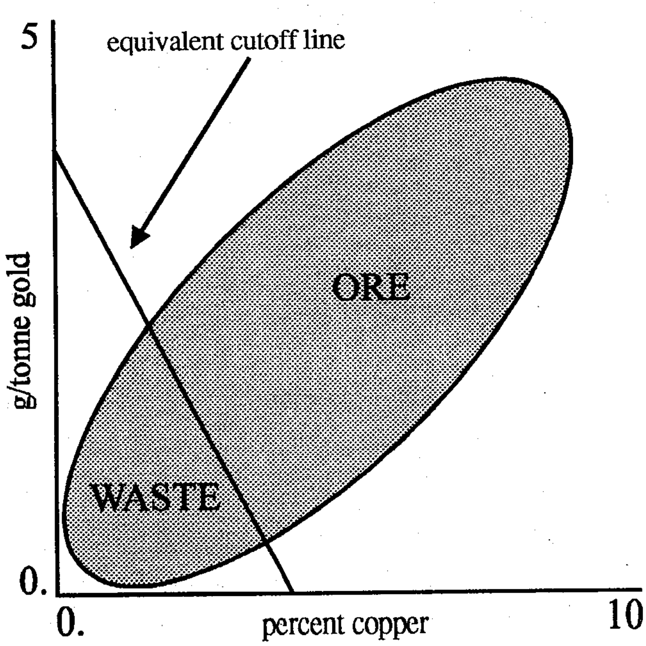

Figure 3
An illustration of a local bivariate histogram of copper and gold block grades in a panel. The bivariate envelope is shown by the shaded ellipsoid. The equivalent cutoff grade is represented by the sloping line on this plot.

Obtaining an estimate of this bivariate histogram can be difficult. The Multigaussian model can provide a practical solution in many cases but the application of Indicator methods is currently difficult and tedious.

PANEL MODELS AND OPTIMIZATION

What are the advantages of using recoverable resource models to estimate resources in a deposit? As presented in this paper, they are

- a more logical approach to the estimation of recoverable resources based
- on mining units compatible with the constraints to be used in production. The excessive smoothing that is inevitably associated with the linear estimation methods leads to an incorrect representation of the distribution of mining block grades.
- estimates which provide a logically correct understanding of the spatial uncertainty in knowing the locations of ore blocks. The panel represents the minimum scale of spatial detail that can be reasonably achieved in the modelling.

These features of the recoverable resource models require some changes to the normal use of the optimization programs. The recoverable resources for each panel for a range of cutoff grades must be represented by a set of tonnages and grades and normally a wide range of cutoffs should be considered. This can be comparatively easily achieved with the Whittle software using the ore type facility; up to 50 different ore types can be defined for each block ore panel. Table 1 present a typical set of incremental resources for a panel for a range of cutoff grades.

Easting	Northing	Elevation	ore type	Cutoff grade	Cubic Metres	Grade
38	24	27	0	0.6 - 0.8	44	0.70
38	24	27	1	0.8 - 1.0	79	0.82
38	24	27	2	1.0 - 1.1	54	1.05
38	24	27	3	1.1 - 1.2	65	1.17
38	24	27	4	1.2 - 1.3	75	1.25
38	24	27	5	1.3 - 1.4	84	1.35
38	24	27	6	1.4 - 1.5	88	1.45
38	24	27	7	1.5 - 1.6	94	1.50
38	24	27	8	1.6 - 1.7	92	1.68
38	24	27	9	1.7 - 1.8	88	1.74
38	24	27	10	1.8 - 1.9	82	1.86
38	24	27	11	1.9 - 2.0	73	1.93
38	24	27	12	2.0 - 2.1	61	2.03
38	24	27	13	2.1 - 2.2	52	2.15
38	24	27	14	2.2 - 50	140	2.45
38	24	27	98	waste	32	0.00

Table 1: An example of the incremental resources for a panel for 15 cutoff grades. This is the general format of the file for the Surpac interface to Whittle optimization (not including the cutoff grades).

The size of the panel presents a significant problem for optimization because of its impact on determining accurate design parameters. What does it mean to use a small block model which allows the determination of accurate engineering constraints and detailed ore scheduling when the individual block grade estimates and the global estimates are grossly in error? For design purposes, the smaller the blocks or panels, the better. In a sense, the need for accurate design parameters must be weighed against the need for accurate estimates of the local recoverable resources.

From the viewpoint of a resource estimation, it seems more sensible to provide the best possible estimates of resources at a local scale and allow for the uncertainties in design. Some initial research supports this conclusion. The problems of estimation error, panel size and pit design are nowhere more apparent than at the bottom of most pits. Pit bottoms often cover a small area, carry relatively high grades (because the pit is driving on those grades) and relatively high uncertainty, and are informed by a small number of drill hole samples compared to higher levels in the pit. It is here that accurate estimation of small block grades is required to design the pit, and yet often the least amount of information is available to achieve this goal.

The currently popular practice of subdividing panels into smaller subblocks with the same grade as the large panel may provide a better looking pit design and create an illusion of confidence but it has no other practical or economic value.

CONCLUSIONS

Traditional block estimation methods including ordinary kriging have serious weaknesses for estimating the grades of small blocks from widely spaced data in resource models. Large block grade estimation errors which result from smoothing and other sources can cause gross errors in local and global resource estimates.

Recoverable resource estimation methods provide an alternative interpretation of block estimation error which can be used to improve resource estimation. Recoverable resource models carry implicit information about the uncertainty in knowing the block grade distribution in a mineral deposit. This information is important for optimization and design of open pits.

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