Option Pricing for Resources Development Projects - Investment Analysis in Post-NPV Era

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Introduction

Price of the commodity is one of the major variables that substantially influence the value of a resource development project. Its uncertainty is of paramount importance in many natural resource industries, where the standard deviation of annual changes in futures prices of relevant commodities sometimes exceeds 40% per year (Bodie, Z and Rosansky, V I 1980). Strategic investment decisions and operation are prerequisite for investors and mine managers. Investors usually have an option to postpone the start of the project till appropriate time, including the option "never to invest" as well, and after its start-up they yet have operating options such as laying-up (mothballing), reopening or abandoning, to cope with the change in economic environment. Effect of these kind of possible strategic managerial responses to price variations or the "skill of the managers" is totally neglected in usual DCF-ROR (discounted cash flow - rate of return) analysis based on NPV (net present value) and discount rates. Having such flexibility in management is equivalent to have an option to get further information and increase possibility to adjust the state

of the project to get better economical results. The value of such option, evaluated using the option pricing techniques developed in the field of financial economics (Black, F. and Scholes, M., 1973), could be rationally incorporated in the evaluation of a project to objectively optimize the strategic investment and managerial decisions.

Postponement Option on Investment

Conventional project evaluation based on NPV analysis as DCF-ROR, always regards that an investment is irreversible and will be executed on a known certain date. But as long as the value of a project, in contrast with the amount of investment, is uncertain, depending on the stochastic changes in the current price of its product, investor always has an option to postpone the decision till appropriate time to get better economical result due to increase of information on economical environment. We expediently call this option a "postponement option" throughout the paper. Comparing this with a stock call option, the amount of investment and the certain date before which the investor has to decide, can be regarded as the exercise price and the maturity date of a call option, respectively. As a call option has a certain option premium depending on current price of the stock and its volatility, and the length of the time till the maturity date, we can easily imagine that a postponement option has a value for itself as well. Therefore, the real value of a project should be the sum of the value of the postponement option (corresponding to an option premium) and the conventional NPV of the project. The conventional NPV of a project is the accumulated NPV of the overall cash flow of the project if it is positive, or zero, if it is negative, so that it could be represented by a bi-linear function of the price of the relevant commodity.

In terms of call option, it seems clear that the higher the price of the stock, the greater the value of the option but less the value of

the option premium. When the stock price is much greater than the exercise price, the option is almost sure to be exercised. The current value of the option will thus be approximately equal to the price of the stock minus the price of a pure discount bond that matures on the same date as the option, with face value equal to the striking price of the option. On the other hand, if the price of the stock is much less than the exercise price, the option is almost sure to expire without being exercised, so its value and also the value of the option premium will be near zero. Considering a resources development project, the higher the price of the current relevant commodity so that the accumulated NPV of the cash in-flow is much larger than the amount of investment, the greater the real value of the project but the less the value of the postponement option as there is virtually no reason to postpone the investment. On the other hand, if the price of the current relevant commodity is very low so that the accumulated NPV of the cash in-flow is far less than the amount of investment, the value of the postponement option again approaches zero as the possibility that the investment be exercised becomes virtually nil. It means that the postponement option plays a great role in project evaluation especially under critical economic conditions.

Contingent claims (option pricing) analysis is based on the assumption that stochastic changes in the accumulated NPV of the cash in-flow of the project E is spanned by existing assets in the economy. In other words the capital markets must be sufficiently complete so that one could find an asset or construct a dynamic portfolio of assets, the price of which is perfectly correlated with E. Following assumptions are made for the analysis:

a) The short-term interest rate is known and is constant through time.

b) The accumulated NPV of the cash in-flow of the project is expressed by a

linear function of the price of the relevant commodity.

c) The commodity price follows a random walk in continuous time with a variance rate proportional to the square of the commodity price. Thus the distribution of possible commodity prices as well as the accumulated NPV of the cash in-flow of the project at the end of any finite interval are log-normal.

d) The project has no dividends or other distributions.

e) There are no penalties to short selling. A seller who does not own a commodity will simply accept the price of the commodity from a buyer, and will agree to settle with the buyer on some future date by paying him an amount equal to the price of the commodity on that date.

The accumulated NPV of the cash in-flow of the project is represented by the commodity price P as,

Equation 1: $\quad E = k(P - P_0)$

where k is a constant and P_0 is the commodity price at which E becomes virtually zero. If P follows the geometric Brownian motion with drift as,

Equation 2: $\quad dP = \mu P dt + \sigma P dz$

where $dz = \varepsilon \sqrt{dt}$, ε following the normal distribution, t, μ, σ are time, expected drift of the commodity price and the volatility of the commodity price, respectively. As E is a linear function of P, E also follows the geometric Brownian motion but with different drift and volatility.

Equation 3: $\quad dE = \mu'E dt + \sigma'E dz$

where $\mu' = \mu \times \dfrac{P}{P - P_0}$ and $\sigma' = \sigma \times \dfrac{P}{P - P_0}$.

Let f be the value of a derivative that only depends on E and t, which means $f(E,t)$. Ito's lemma gives the differential df as

Equation 4:

$$df = \frac{\partial f}{\partial t} dt + \frac{\partial f}{\partial E} dE + \frac{1}{2} \frac{\partial^2 f}{\partial E^2} (dE)^2$$

Substituting Equation 3 for dE, we obtain their expanded forms as

Equation 5:

$$df = \left(\frac{\partial f}{\partial E} \mu E + \frac{\partial f}{\partial t} + \frac{1}{2} \frac{\partial^2 f}{\partial E^2} \sigma^2 E^2 \right) dt + \frac{\partial f}{\partial E} \sigma E dz$$

Equation 3 and Equation 5 have the same Wiener process, namely $dz = \varepsilon \sqrt{dt}$, which represent the risk term. Therefore, considering proper dynamic portfolio, consisting of the project and a derivative, it is possible to create a hedged (riskless) position, whose value will not depend on the price of the commodity, but will depend only on time and the values of known constants. The number of derivatives that must be sold short against the project long is

Equation 6: $\quad \dfrac{1}{\dfrac{\partial f(E,t)}{\partial E}} = \dfrac{k}{\dfrac{\partial f(E(P),t)}{\partial P}}$

To see that the value of such a hedged position does not depend on E nor P, note that the ratio of the change in the value of a derivative to the change in E, when its change is small, is $\dfrac{\partial f(E,t)}{\partial E}$.

To a first approximation, if E or P changes by an amount $\Delta E = k\Delta P$, the value of a derivative will change by an amount

$\dfrac{\partial f(E,t)}{\partial E} \Delta E$ or $\dfrac{\partial f(E(P),t)}{k \partial P} k\Delta P$ and the number of derivatives given by expression 6 will change by an amount ΔE or $k\Delta P$. Thus, the change in the value of a long position of the project will be approximately offset by the change in the value of a short position in

$\dfrac{1}{\dfrac{\partial f(E,t)}{\partial E}}$ or $\dfrac{k}{\dfrac{\partial f(E(P),t)}{\partial P}}$ units of derivatives.

Since the hedged position contains the project long and $\dfrac{1}{\dfrac{\partial f(E,t)}{\partial E}}$ units derivatives short, the value of such asset in the position Π is

Equation 7: $\quad \Pi = E - \dfrac{f}{f_E}$

where f_E represents $\dfrac{\partial f(E,t)}{\partial E}$. The change in the value of the asset $d\Pi$ in a short interval dt is

Equation 8: $\quad d\Pi = dE - \dfrac{df}{f_E}$

Substituting Equation 3 and Equation 5, we obtain:

Equation 9:

$$ d\Pi = -\left(\frac{1}{f_E}\right)\left(\frac{1}{2}f_{EE}\sigma^2 E^2 + f_t\right)dt $$

Since the return on the asset in the hedged position is certain, the return must be equal to rdt, where r denotes a riskless interest rate. Thus the change in the value of the asset Equation 9 must equal the value of such asset times rdt.

Equation 10:

$$ -\left(\frac{1}{f_E}\right)\left(\frac{1}{2}f_{EE}\sigma^2 E^2 + f_t\right)dt = \left(E - \frac{f}{f_E}\right)rdt $$

Dropping the dt from both sides, and rearranging, we have the so-called Black and Scholes differential equation for E and derivatives.

Equation 11: $\quad f_t + rEf_E + \dfrac{1}{2}f_{EE}\sigma^2 E^2 = rf$

Writing T for the certain date at which the investor has to decide whether to execute the investment or not.

Equation 12: $\quad f(E,T) = \max(E - I, 0)$

where I denotes the amount of investment to kick off the project. Solving Equation 11

for f, subject to the boundary condition Equation 12 gives the real value of the project that includes the value of the postponement option.

Equation 13: $\quad f = EN(d_1) - Ie^{r(t-T)}N(d_2)$

$$ d_1 = \frac{\ln(E/I) + \left(r + \dfrac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}} $$

$$ d_2 = \frac{\ln(E/I) + \left(r - \dfrac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}} $$

As we neglect the opportunity cost of delaying construction of the project, f increases monotonously according to the postponement of the maturity date T. Under more realistic circumstance where delaying construction of the project has some opportunity cost such as royalty, we can determine the optimum date to exercise the investment.

Operating Options

Once a resources development project has been started, manager has to follow an optimum operating strategy other than production rate or cut-off grade. There are several operating options such as laying-up (mothballing), reopening or abandoning, to cope with the change in economic environment. Considering these options, the present value of a project becomes larger than the value, calculated by conventional NPV analysis based on expected cash flow, as long as the project is optimally managed.

Here we briefly summarize the framework proposed by Brennan and Schwartz (1985). Again the price of a commodity P is assumed to follow a geometric Brownian motion with drift as in Equation 2 and let $F(P,\tau)$ represent the futures price at time t for delivery on one unit of the commodity at time T where $\tau = T - t$.

Then using Ito's lemma the differential of F becomes

Equation 14:

$$dF = \left(-F_\tau + \frac{1}{2} F_{PP} \sigma^2 P^2 \right) dt + F_P dP$$

A commodity usually has a convenience yield C, which is defined as a flow of services that accrues to an owner of a physical commodity and assumed to depend only on P, thus $C(P)$. Assuming a portfolio consisted of $\frac{1}{P}$ commodity and $\frac{1}{P \frac{\partial F}{\partial P}}$ of its futures, the value of this portfolio at present Π_1 is,

Equation 15: $\quad \Pi_1 = \frac{1}{P} P - (P F_P)^{-1} F = 1$

and the change of its value during dt is

Equation 16: $\quad d\Pi_1 = \frac{dP}{P} - (P F_P)^{-1} dF$

The owner of this portfolio earns the sum of $d\Pi_1$ and the marginal net convenience yield $\frac{C(P) dt}{P}$ during dt without any risk. Thus, this return must be equal to the riskless return $r\Pi_1 dt = r dt$ and we obtain

Equation 17: $\quad -F_\tau + \frac{1}{2} F_{PP} \sigma^2 P^2 = -F_P (rP - C)$

Substituting Equation 17 to Equation 14, the instantaneous change in the futures price may be expressed in terms of the convenience yield and the instantaneous change in the spot price as

Equation 18: $\quad dF = F_P [P(\mu - r) + C] dt + F_P P \sigma dz$

The value of the resources development project H depends on the current commodity price P, the physical inventory in the deposit Q, calendar time t and the operating policy ϕ. Thus,

Equation 19: $\quad H \equiv H(P, Q, t; j, \phi)$

The indicator variable j takes the value one if the project is in operation and zero if it is laid-up or abandoned. In case of a mine, the operating policy is described by the function determining its production rate $q(P, Q, t)$, and three critical commodity prices $P_1(Q, t)$, $P_2(Q, t)$, $P_0(Q, t)$, respectively representing the prices at which the mine is closed down or abandoned if it was previously open, the mine is opened up if it was previously closed, and the mine is abandoned if it is already closed. Applying Ito's lemma to Equation 19, the instantaneous change in the value of the project is given by

Equation 20:

$$dH = H_P dP + H_Q dQ + H_t dt + \frac{1}{2} H_{PP} (dP)^2$$

The after-tax cash flow from the project is

Equation 21: $\quad q(P - A) - M(1 - j) - \lambda_j H - T$

where, $A(q, Q, t)$ is the average cash cost rate of producing at the rate q at time t when the inventory is Q $M(t)$ is the after-tax fixed-cost rate of maintaining the mine when it is closed and open

λ_j ($j = 0,1$) is proportional rate of tax on the value of the mine when it is closed and open

$T(q, Q, P, t)$ is the total income tax and royalties levied on the mine when it is operating

Assuming that $\frac{H_P}{F_P}$ futures of the commodity be sold short against a unit project long, the owner of such portfolio earns the sum of the change in its value,

Equation 22: $\quad d\Pi_2 = dH - \frac{H_P}{F_P} dF$

and the cash flow as Equation 21 during dt as

Equation 23:

$$\left(\frac{1}{2}\sigma^2 P^2 H_{PP} - qH_Q + H_t + q(P-A) - M(1-j) - T - \lambda_j H + (rP-C)H_P\right)dt$$

without any risk, if we ignore the possibility of expropriation. To avoid riskless arbitrage opportunities, this return must be equal to the riskless return on the value of the investment $rHdt$. Thus we obtain

Equation 24:

$$\frac{1}{2}\sigma_2^2 P^2 H_{PP} + (rP-C)H_P - qH_Q + H_t + q(P-A) - M(1-j) - T - (r+\lambda_j)H = 0$$

Under the value maximizing operating policy $\phi^* \equiv \{q^*, P_0^*, P_1^*, P_2^*\}$, the values of the operating project and mothballed project are respectively given by

Equation 25 (a):

$$V(P,Q,t) \equiv \max_{\phi} H(P,Q,t,1,\phi)$$

Equation 25 (b):

$$W(P,Q,t) \equiv \max_{\phi} H(P,Q,t,0,\phi)$$

Let $\bar{q}$ and $\underline{q}$ the upper and lower limits of production rate during the operation of the project, the value of the project under the value-maximizing policy satisfy the following two equations.

Equation 26 (a):

$$\max_{q \in \{\underline{q},\bar{q}\}}\left[\frac{1}{2}\sigma^2 P^2 V_{PP} + (rP-C)V_P - qV_Q + V_t + q(P-A) - T - (r+\lambda_1)V\right] = 0$$

Equation 26 (b):

$$\frac{1}{2}\sigma^2 P^2 W_{PP} + (rP-C)W_P + W_t - M - (r+\lambda_2)W = 0$$

The value of the project depends on time only because the costs A, M, C and some miscellaneous costs like costs for closing and reopening the project $K_1(Q,t)$ and $K_2(Q,t)$ depend on time. Assuming a constant rate of inflation π in all of these and $C(P,t) = \kappa P$ then Equations 26 may be

simplified, using following deflated variables,

$$a(q,Q) = A(q,Q,t)e^{-\pi t} \; ; \; m = M(t)e^{-\pi t},$$

$$k_1(Q) = K_1(Q,t)e^{-\pi t} \; ; \; k_2(Q) = K_2(Q,t)e^{-\pi t}$$

$$p = Pe^{-\pi t} \; ; \; v(p,Q) = V(P,Q,t)e^{-\pi t} \; ;$$

$$w(p,Q) = W(P,Q,t)e^{-\pi t}$$

as Equations 27.

Equation 27 (a):

$$\max_{q \in \{\underline{q},\bar{q}\}}\left[\frac{1}{2}\sigma^2 p^2 v_{pp} + (\rho-\kappa)pv_p - qv_Q + q(p-a) - T' - (\rho+\lambda_1)v\right] = 0$$

Equation 27 (b):

$$\frac{1}{2}\sigma^2 p^2 w_{pp} + (\rho - \kappa)pw_p - m - (\rho + \lambda_2)w = 0$$

where, $\rho = r - \pi$ is the real riskless interest rate.

T' is the real total income tax and royalties levied on the mine when it is operating.

Case Study of a Copper Mine

Postponement Option

We applied the option pricing for a small copper mine project in South America to compare with usual NPV analysis. The production of the mine started in 1989 after three years of initial investment. The scheduled project life was ten years from the first year of production. We expediently accumulate the amount of initial investment and assume the whole investment was done in 1988 and set 1987 as present. Thus, all the values of the following cash in-flow are discounted to the year 1987 to obtain the present value of E. The rate of inflation is based on 20 years average of GDP deflator. Also a 20 years average of US treasury bill rate for 3 months is assumed as a riskless interest rate to eliminate the strong inflationary pressure in US during late 1970's to the beginning of 1980's. By-product is converted to copper assuming its price perfectly correlating with that of copper. The parameters used for the

analysis are contained in Table 1. Average copper price in 1987 was 1,820$/ton in US and the accumulated NPV of the cash in-

flow of the project E at that copper price becomes $2,804,471.

Rate of inflation	5.93%
Riskless interest rate	7.45
Discount rate for E	15.0%
Average copper price in 1987	1,820$/ton
Copper price at which E becomes zero: P_0	1,580$/ton
Volatility of copper price σ	0.15
Amount of investment I	$2,231,047
Volatility of E	1.16

Table 1: Data for a Small Copper Mine Project

The value of the project, including the value of the postponement option, monotonously increases according the

increase of the length of grace (length of the period in which the investor can post pone the decision) as shown in Figure 1.
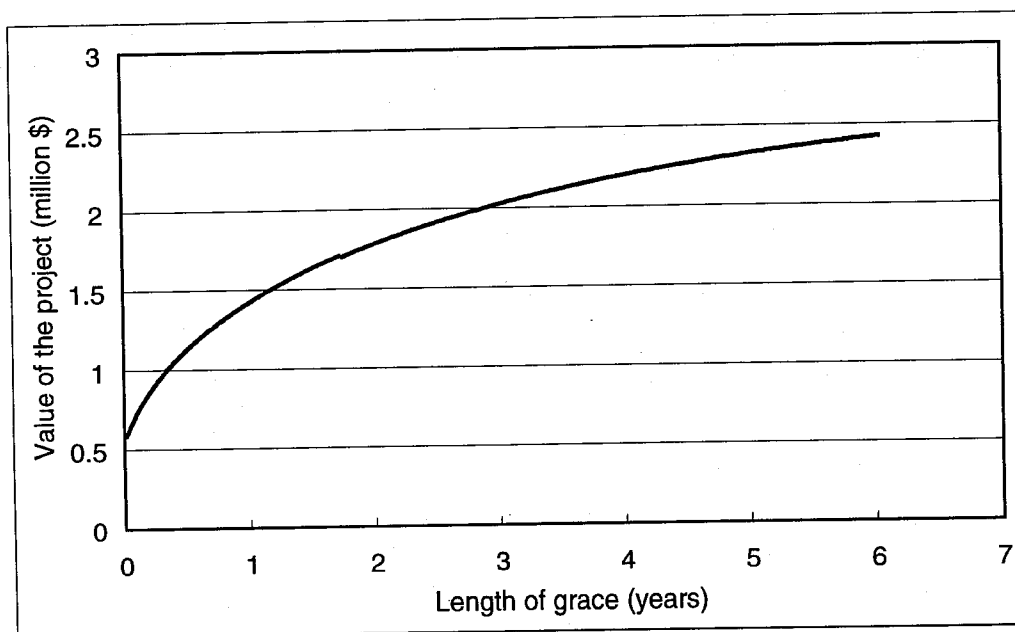
Figure 1: Value of the Project vs. Length of Grace

But considering an opportunity cost for the delay of the project as royalty or interest on loans, we can optimize the date investor should make his decision. Figure 2 shows the change in value of the project according to the length of grace, considering an opportunity cost of 0.5 million $/year, which increases year by year at the rate of inflation.
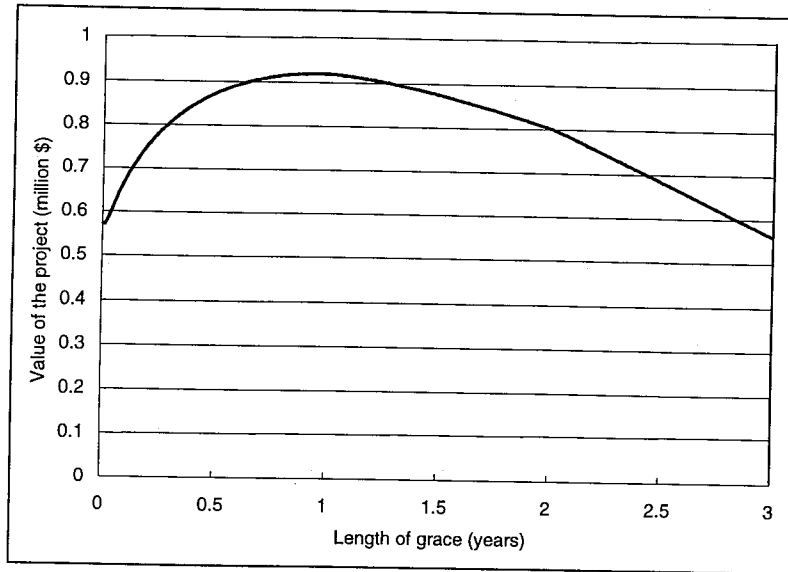

Figure 2: Value of the Project Considering Opportunity Cost

The optimum date, investor should make his decision under this condition, is approximately after one year (more exactly 0.96 years). Figure 3 shows the comparison between the values of the project calculated by conventional method and option pricing. The length of grace is set at 0.96 years for the calculation.
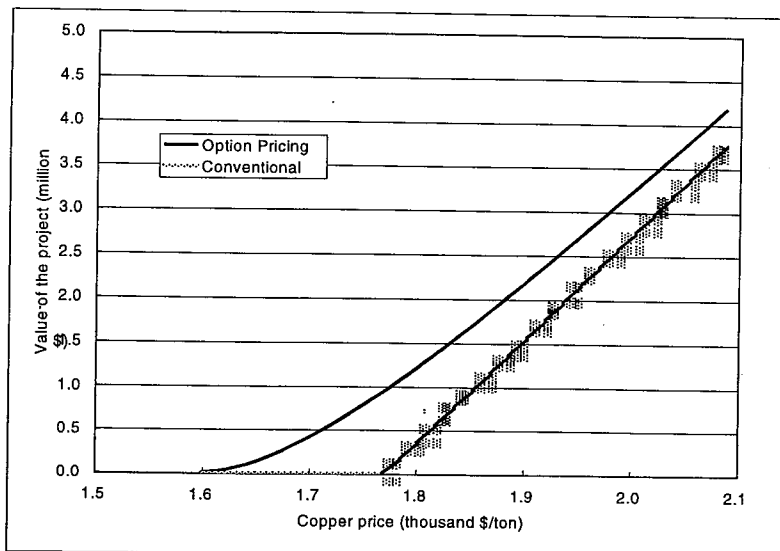

Figure 3: Value of the Project Including Postponement Option

Operating Options

The real value of the project, considering the operating options, under the value-maximizing policy is the solution of Equations 27 subject to proper boundary conditions. Conventional NPV of the project has also been obtained. To compare with the real value of the project, the rate of inflation has been used as the discount rate. All the evaluation is done as 1987 as present. We used Equation 28 for tax T' and neglected λ_j.

Equation 28: $T'(q, p) = t_1 qp + t_2 q[p(1 - t_1) - a]$

where,

t_1 is the royalty rate and t_2 is the income tax rate. Depreciation tax allowances are ignored.

Assuming $v_Q = \dfrac{v}{Q}$, which means that the value of the project linearly correlates with the amount of inventory, Equation 27 (a): is simplified as

Equation 29:

$$\frac{1}{2}\sigma^2 p^2 v_{pp} + (\rho - \kappa)p v_p - \left(\rho + \frac{q}{Q}\right)v + q(p - a) - T' = 0$$

Also assuming q, k_1, and k_2 as constant, Equation 29 and Equation 27(b) can be solved as an ordinary differential equations of p subject to following boundary conditions:

$w(p_0) = 0$

$v(p_1) = w(p_1) - k_1$

$w(p_2) = v(p_2) - k_2$

$v_p(p_1) = w_p(p_1)$

$v_p(p_2) = w_p(p_2)$

The general solution for Equation 29 and Equation 27(b) is

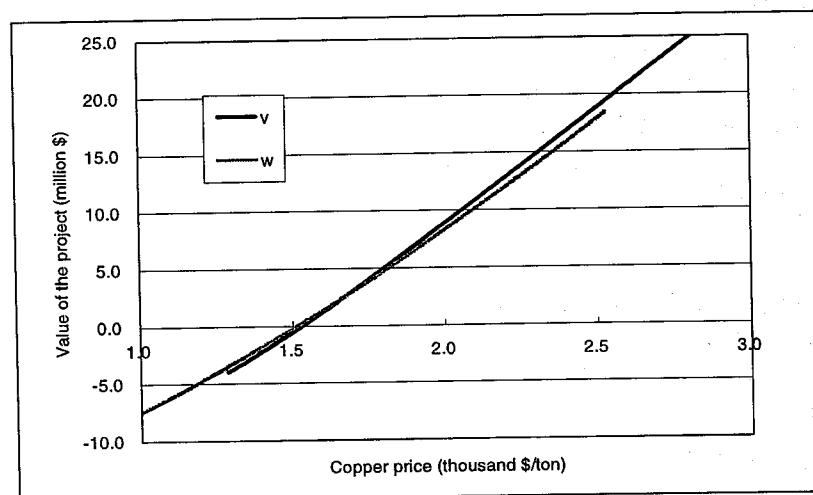**Equation 30(a):** $v(p) = \beta_3 + \beta_4 p^{\gamma_3} + \beta_5 p^{\gamma_4} + \beta_6 p$

Equation 30(b): $w(p) = \beta_0 + \beta_1 p^{\gamma_1} + \beta_2 p^{\gamma_2}$

where,

$\gamma_1 > 1, \ \gamma_2 < 0 \qquad \gamma_3 > 1, \ \gamma_4 < 0$

To avoid divergence at both ends of the price 0 and ∞, both β_2 and β_4 must be zero.

In the case of the same South American copper mine following $v(p)$ and $w(p)$ as shown in Figure 4. Copper price at the beginning of 1988 is assumed as 1,818.8 $/ton. m, k_1, k_2 are assumed as 0.25, 0.5, 1.0 million $, respectively.

Figure 4: Value of the Project Considering Operating Options

The real value of the project including the value of the operating options at the beginning of 1988 is 5.32 million $, where as the conventional NPV discounted by the rate of inflation is 4.36 million $. Thus, the option premium under this condition is 0.96 million $. p_0, p_1, p_2 are 1,520, 1,290, 2,530 $/ton, respectively. This project is found out to be very critical so that $p_0 > p_1$, which means if the price drops below 1,520 $/ton, the mine should be abandoned rather than mothballed.

Conclusion

Effect of possible strategic managerial responses to price variations such as postponing the decision to start-up a project, laying-up (mothballing), reopening or abandoning a running project to cope with the change in economic environment, usually recognized as the "skill of the managers", could successfully be incorporated in the evaluation of a project. In the case of small copper mine project in South America, which is very critical based on conventional NPV analysis, considerable option premium both for postponement option and operating options could be recognized. But at the same time, it was also found out that under the set condition, there is virtually no option for mothballing.

Acknowledgements

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