Optimizing Conditionally Simulated Orebodies with Whittle 4D
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Abstract

The application of the Lerchs-Grossmann pit optimizing algorithm to an interpolated grade model is common throughout the open pit gold mining industry. This is done to assess the economic sensitivity of mining of a given gold deposit to operating and capital costs, slope constraints, and processing plant performance. Although global adjustments to grades (or metal values) are possible within Whittle 4D, this procedure cannot be uniquely applied to individual blocks to assess the economic sensitivity due to block grade uncertainty.

Conditional simulations provide multiple, equiprobable realisations of the target variable which honours, on a global basis, the univariate statistics of the original data, as well as its spatial continuity, as described by the variogram.

Under an uncertainty model based on conditional simulations, it is possible to assess the sensitivity of the overall pit economics, long term mine plan, and mine schedules to grade uncertainty. This can be done at a "local" scale, ie, by benches or mining phases. In this case, the Whittle 4D algorithm is seen as a Transfer Function that can be used to translate equiprobable block models into alternative optimized pits, mine plans, and schedules.

Introduction

In the mining industry block models are typically developed as tools used to estimate the overall resources of a deposit, and to allow further processing for planning the extraction and mine schedule for the life of the mine. The Lerchs-Grossmann 3D Graph Theory algorithm (Lerchs and Grossmann, 1965), implemented in Whittle 4D as the Nested Lerchs-Grossmann algorithm (Whittle, 1988), is possibly the most popular tool used to develop an optimized pit, from which further refinements can be made at a "local" scale to account for certain practical mining considerations.

During this process a mine schedule, characterized usually by "mining phases", is also developed. The final result is a mine plan that describes an "optimized" scenario of how the orebody would be best mined in an economic sense, with due consideration to technical aspects of mining the deposit. It is generally accepted that the block model reserve, estimated from exploration or development drill holes, and mine production prediction (based on blastholes) frequently show significant discrepancies (Rossi and Parker, 1993). Further, if reconciliation is done against production figures, the discrepancies are even larger. This usually results in deviations from the economic "optimum" pit developed at a given planning stage, in some cases large enough to cause the whole operation to become uneconomic.

For evaluation and planning purposes, it is desirable to understand the source of these errors and to minimise these discrepancies.

Many are the possible reasons for such discrepancies, such as the change of support effect, insufficient drilling, bad sampling or preparation techniques, interpolation errors in building the block model, the smoothing effect of the moving-average interpolators in vogue, etc. (Rossi et al., 1993).

However, regardless of the reasons, it is important to recognise that there are no "perfect" block models, in the sense that the grades and tonnages described have a measure of error (uncertainty).

Until recently, although many in the industry recognised the limitations of the interpolated block models, there was little that could be done in practice to assess its uncertainty, particularly on a block-by-block basis. Propositions such as using the kriging variance to obtain confidence intervals for individual blocks have failed miserably (Srivastava, 1987, and Journel, 1988, among others), and the best we could hope for was a "global" assessment of uncertainty, where the overall deposit grade and tonnages above cutoffs are given a confidence level (Journel and Huijbregts, 1978). However, on a smaller scale (monthly production, for example), this could not be accurately done. In addition, it should be noted that even if we had a "reliable" estimation variance, or the complete cumulative distribution for each block grade was known, this would not provide us with any measure of uncertainty about the minable reserves after optimizing with Whittle 4D. In theory, the optimization process would have to be linear if confidence intervals obtained during the block model construction are to be used as confidence intervals for minable reserves (post Whittle 4D).

In the last 10 years or so significant developments have been made in the area of geostatistical conditional simulations (Alabert, 1987; Journel, 1988; Isaaks, 1990). Theoretical developments, in addition to the availability of cheaper and more powerful hardware and software (Deutsch and Journel, 1992), have allowed practitioners to begin implementing large scale orebody simulations. These simulations have been used to date (with few exceptions) as either grade control tools, or to develop a "ground truth" image of the mineralisation at a small scale (Rossi et al, 1993 and 1994). In this paper, a case study is described where the simulations are used to assess block-by-block uncertainty, and Whittle 4D is used to translate that uncertainty into minable (recoverable) reserves. This allows for a better sensitivity analysis of the economic and technical aspects of the operation.

Conditional Simulations and Models of Uncertainty

The purpose is to build a model that honours the full histogram and variogram of the conditioning data, and therefore honours the spatial variability of the deposit as represented by the conditioning data. Blast hole data should be used for conditioning whenever possible, although it is possible to build such a model from exploration data alone. It is also feasible to directly incorporate geologic features into some models, depending on the simulation algorithm used, or by simulating the geologic characteristics of the deposit themselves. All conditional simulations are typically built on very fine grids, as fine as possible given the hardware available. By honouring the histogram, the model correctly represents the proportion of high and low values, the mean, the variance, and other statistical characteristics of the data. By honouring the variogram, it correctly portrays the spatial complexity of the orebody, and the connectivity of low and high grade zones. These are fundamental pieces of information at the design, planning, and scheduling phases of the project. When several simulated models are obtained, the final product is a good basis for assessing recoverable reserves and its uncertainty.

A reasonable grid for the simulation is between 1ft by 1ft to 3ft by 3ft. Larger grid sizes (up to 10ft by 10ft, as in the Case Study described below) are used sometimes because of the amount of computer time and hard disk space involved. Such a high resolution is possible because of the random aspects of the algorithms used (conditional simulations are Monte-Carlo-based techniques). The data used to condition the simulation have to belong to the same population. Shift in the attitude of the ore controlling structures requires the separation of the data in different populations, as would geologic or lithologic boundaries. Thorough knowledge of the behaviour of the high-grade population is required to control high grades in the simulation, refer to Parker (1991). Issues such as limiting the maximum simulated grade should be carefully considered.

The simulation method should be decided based upon the type of the deposit, the available data set and the desired output. The first decision is whether to use a parametric or non-parametric approach. Examples of each are the Sequential Gaussian (Isaaks, 1990) and Sequential Indicator (Alabert, 1987) simulations. The latter is more complicated, based on multiple indicator kriging techniques (Journel, 1988), and requires definition of several indicator cutoffs. The former is simpler and quicker, although more restrictive in its basic assumptions. Any available geological criteria ("soft" information, see Journel, 1986, or Deutsch and Journel, 1992) should be used. As with any estimation exercise, variograms should be estimated and modelled, and a number of other important parameters need to be considered. These include: minimum and maximum data value and simulated value allowed, number of conditioning data to be used, search distances, anisotropies, etc.

Finally, it is very important to thoroughly check the simulated values. The histogram of the simulated data should be compared to that of the original conditioning data. Both should be similar in terms of simple statistics and overall shape of the histogram. The variograms of the simulated data should be similar to the input models, at least up to the search distance used. The original conditioning data and the simulated data should be plotted on maps at the same scale. Close examination of the two sets of maps will detect any possible deviations of the simulated data from the conditioning data. It is also instructive to observe what the conditional simulation produces in areas where there is little or no original (conditioning) data.

When a number of these conditional simulations have been run and checked, then, for each block defined in the grid, there are the same number of possible grades available. These set of grades, all equiprobable by construction, are interpreted to describe the model of uncertainty for that block, generally arranged as a posterior cumulative conditional probability curve. Preferably, a large number of simulations are needed to describe this curve better; however, due to practical limitations, a much smaller number can be used as an initial approximation.

Whittle 4D Optimization as a Transfer Function

In this paper, the overall process of estimating a block model, and performing mine planning and scheduling is seen as an integrated process. Consequently, the Whittle 4D optimization and posterior mine planning and scheduling work is seen as a "transfer function" (Journel, 1988) that represents key aspects of the mine feasibility study, or further mine development after the open pit is in operation. Since it is recognised that the block models always carry uncertainty, a "perfect" input of block grades into Whittle 4D is not possible. Hence, the idea is to input each of the simulations described above (or a subset of them, selected according to given criteria relevant to the problem under investigation) to obtain a "response distribution", ie, a series of "optimal" pits, all equiprobable, that can be used to assess the impact of possible variations in grade at a local scale.

For example, applications of this concept may include the need to assess the uncertainty of the block grades near the bottom of the proposed pit. Often open pits are driven downwards by a few high grade composites, which are responsible for producing an area with high grade block estimates, which in turn pay off extended push-backs and digging to deeper levels of the deposit. A good understanding of the risks involved is necessary to avoid surprises, particularly if the deposit is marginal, and the issue has to be resolved before capital investment or mine development takes place.

Another example could be that for certain periods of the mine life, the mine plan calls for extended periods of mostly waste mining (stripping). The mill or heaps are planned to be fed with ore previously mined and stockpiled. In such case, an ore short fall could develop, and it would be important to know how much risk is involved if either the grades stockpiled are not the planned grades, or if the grades found after completing the push-back are not as planned, or if there could be more waste material than planned that has to be removed before the open pit can extract new ore grade material. In general, almost every mine has a mine plan that "tightens" at certain periods of time. This calls for a detailed (and local) assessment of grade uncertainty.

Therefore, despite the increased labor and effort of this procedure, it appears that the availability of alternative simulated block models (images of grade distribution) can have significant impact in improving pit optimization, mine plans and schedules, and overall analysis of the mine economics.

CASE STUDY - The Database

The database used in this case study was derived from an existing Echo Bay gold mine operating in Nevada. An adjustment has been made to the grades so that the results do not reflect directly upon the current life of mine plan.

The open pit mine operates at 200,000 tpd, mining on 35' high benches. The operation utilises 28 yd^3 electric shovels supported by CAT 992 and 994 loaders. Haulage units are primarily CAT 785 and

789 mechanical drive trucks. The mine feeds three processes, utilising two separate crushing streams, and a run of mine feed. The processes include an 8,000 tpd mill (MILL), an asphalt based, crushed ore, off loadable leach pad (RPAD), and an uncrushed dedicated leach pad (DPAD). Crushing capacity to the RPAD it is 40,000 tpd. All processes are currently operated using fixed cutoff grades.

The deposit is volcanic hosted, straddling the margin of a collapsed caldera. At least three relatively flat lying tuffaceous units are mineralised as well as the underlying metasediments. Gold mineralisation is more often found along fracture surfaces in the more brittle tuffaceous units, and disseminated into open pumice sites in the less densely welded units. Gold mineralisation is fracture controlled in the metasediments. The deposit mined to date has been predominantly oxide material, with a significant sulfide component remaining at depth.

The lower grade oxide material is readily heap leachable since the gold is either exposed along fractures which form a natural break during mining or easily located by cyanide solutions in open pumice sites. Sulfide material is the primary mill feed.

Echo Bay Mines provided a composited drillhole database, a kriged block model, and the Whittle parameter files used for 4D optimization, for use in this study.

Univariate Statistics and Variography by Rock Type

Most of the work required to prepare a conditional simulation, in terms of exploratory data analysis and variography, was already done when preparing the block model currently in use. The main task in this case was the application of the normal transform (anamorphosis) to the composites to transform the original univariate distribution into a normal distribution.

The gold grade simple univariate statistics were run on the database described above. Histograms, overall and by rock types, were created. The relationship between Au grade and lithology was also analysed, both through statistics and spatially, from the cross sections and bench maps available. In some limited cases, the original rock types were re-grouped, mostly on the basis of similar statistics and number of composites available. This re-classification of lithologies did not impact the major (gold-bearing) rock types, but some marginal lithologies, mostly outside the area of main interest, and with a limited spatial extent.

Variography was also done separately on a rock type basis, on the transformed data. The main directions of anisotropy were taken from the previous variography work, as well as angle of tolerances. Usually, variography is a most demanding task, although in this case some of the preliminary analysis had been done at the time of creating the kriged block model.

Conditional Simulations

A group of Sequential Gaussian Simulations (SGS, Isaaks, 1990) were run on this composite database. The simulations were also done separately for each rock type defined above. Many of the parameters required in the simulations were derived from previous work, including search ellipsoids, directions of anisotropy, variography, etc. Two small trial simulations were created to analyse the effect of some of the assumptions and parameters utilised.

A total of five simulations were obtained. These simulations were satisfactory in the sense that they reproduced the original shape of the histograms and, therefore, the overall and conditional statistics of the data, particularly around the economic cutoffs of interest. Both the input histograms and the variograms should be similar to the simulated histograms and variograms (not presented here). This is done in order to ensure that the input data (composites and statistical characteristics) are reproduced in the simulations. In general, this reproduction of the original statistics is easier to achieve in the areas where the data is more abundant. In the least conditioned areas (outside the main zone of interest), the simulations will have poorer reproduction of histograms and variograms. In this case study, some of the peripheral areas of the deposit (outside the largest pit optimized using the kriged block model) were removed so that the final comparison would be more appropriate.

Whittle 4D Optimization

The Whittle 4D analysis program package contains many built in options to measure the sensitivity of the value of an ultimate pit to the input parameters. However, no direct analysis is available to treat the inherent uncertainty in the block grades themselves as represented by metal content in the input model file. A work around does exist for this problem, by simultaneously varying the mining dilution factor and the mining recovery factor, any percentage of the contained metal value can be used in the optimization.

For example:

110% Metal = 0.91 Mining Dilution Factor * 1.1
Mining Recovery Factor = 100% Tons

In this equation, the mining recovery factor is used to control the quantity of metal in all blocks, and the mining dilution factor corrects the tonnage so that total block tonnage does not vary from the original value. The choice of the percentage adjustment is an arbitrary one for the user, and not likely to be based upon the variance of the metal distributed throughout the orebody.

A total of 35 pit shells were examined for the kriged estimate, five separate sequential Gaussian simulations, and the 90% and 110% metal adjusted kriged models. Discounted cash flow analysis of the different scenarios is presented graphically in Figures 1 and 2. Figure 1, showing only the kriged estimate and the adjusted models, suggests that although the initial pits in the sequences have a variation in cash flow, ultimately the difference in discounted cash flow is nearly unaffected by the possible grade variation. Figure 2, comparing the kriged estimate to the simulations, confirms that the discounted cash flows from the initial pits may be variable. In addition, Figure 2 shows clearly that there is a 2 out of 5 (40%) chance that the discounted cash flow for the ultimate pit may be significantly less than originally planned.

Figures 3 and 4 address mill feed release, and Figures 5 and 6 show the grade of that release. Figure 3, showing only the kriged estimate and the adjusted models, suggests that there may be a mill production short fall below 8,000 tpd during the middle to later years of the mine life depending upon the scenario examined. Figure 4, comparing the kriged estimate to the simulations, suggests that a stockpile would be beneficial in the early years of the mine life, but that available mill feed is likely to be considerably greater than that predicted by the kriged estimate over the mine life. Concurrently, the anticipated mill feed grade in Figure 5 is shown to be marginally conservative in Figure 6.

Detailed results of the optimizations are summarized in Table 1.

Conclusions

Undoubtedly, the future of pit optimization and mine planning lies in a more integrated approach, that would encompass long-term estimation, short-term models, and production planning, on a local and global scale, for project development and operating mines alike. Conditional simulation is a tool that allows for a wide variety of optimization and engineering studies in the reserve and mine planning area, and in particular, resolves the inherent shortcomings of other recoverable reserve estimation methods currently in vogue.

The use of Whittle 4D in conjunction with conditional simulation is thus recommended. Its main advantage being that pit optimization of multiple realisations of the mineralisation in an orebody produces a directly quantifiable measure of the risk associated with a mine plan.

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The interpretations and opinions expressed herein are solely those of the authors.

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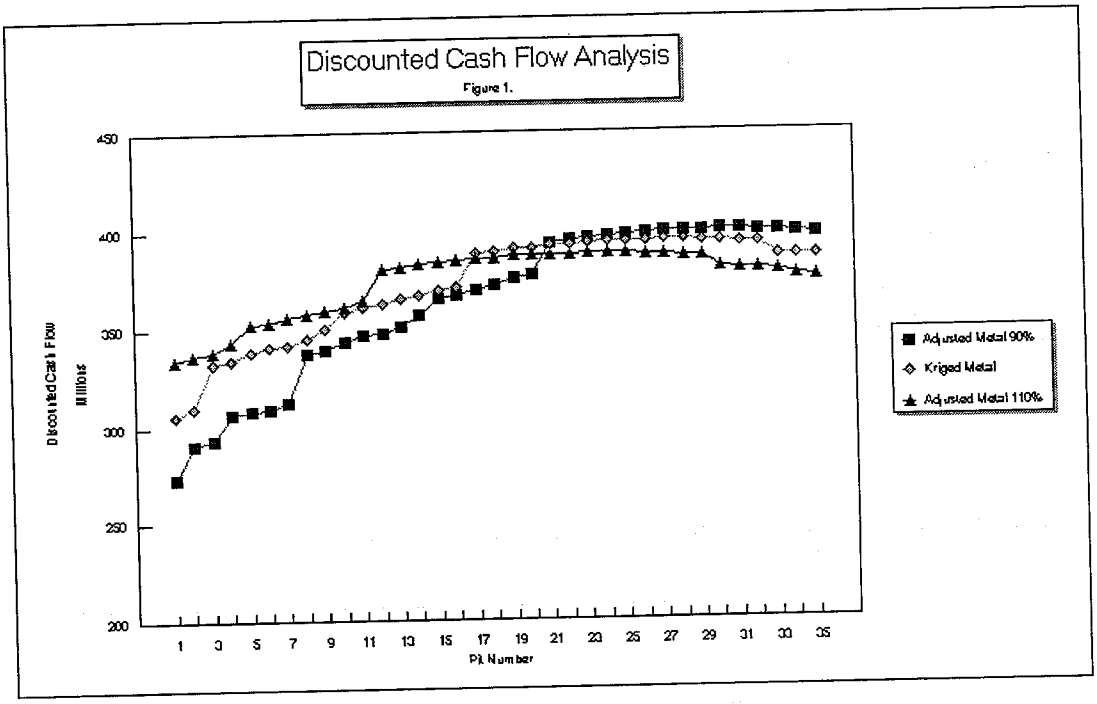
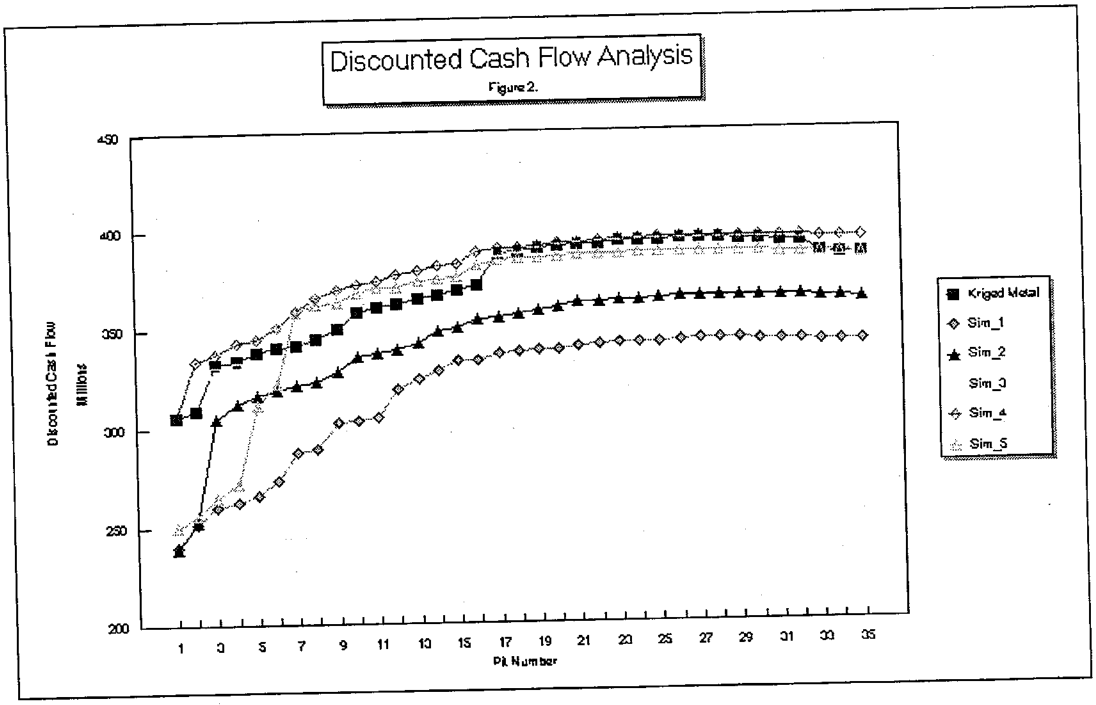
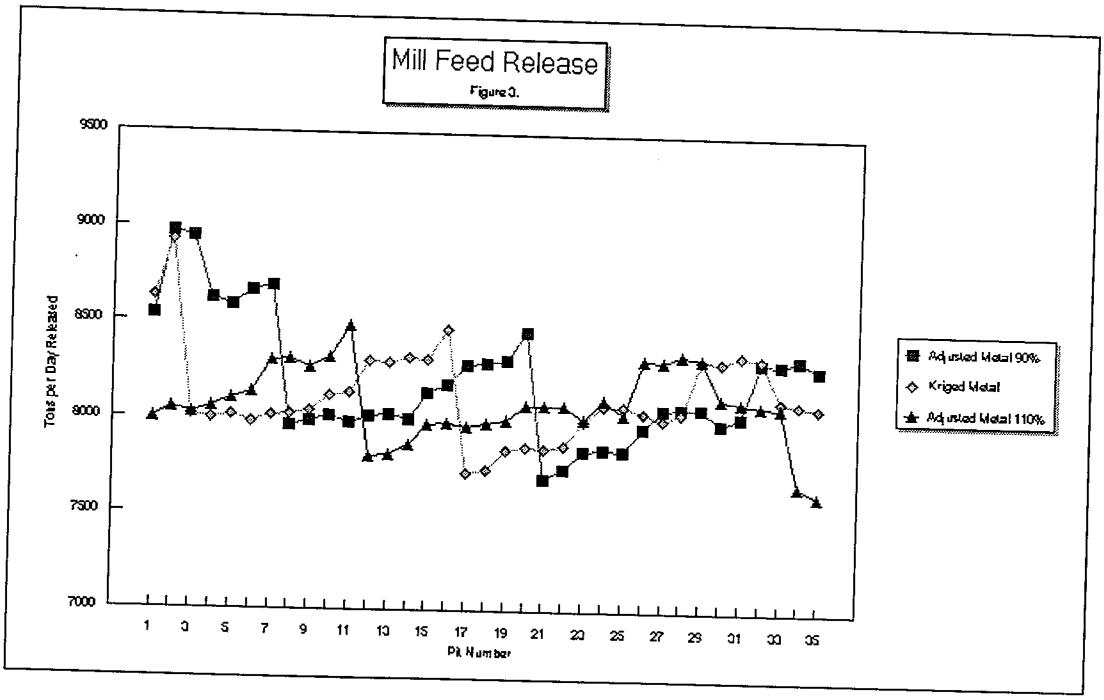
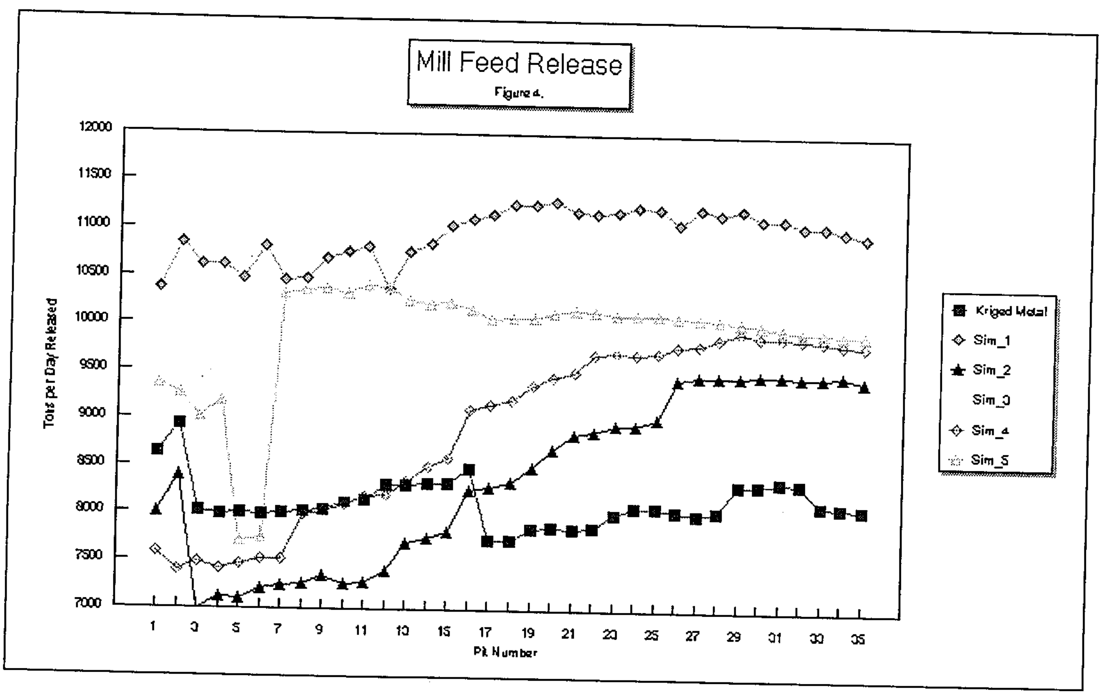
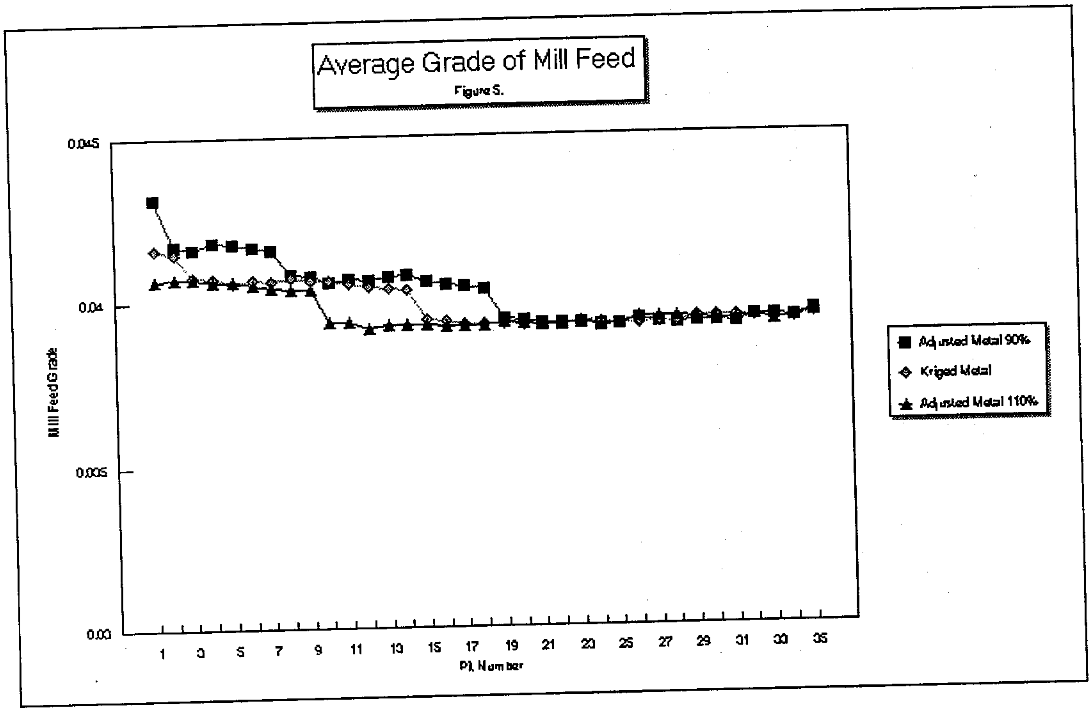

Discounted Cash Flow Analysis
Figure 1.
Discounted Cash Flow
Millions
450
400
350
300
250
200
1 3 5 7 9 11 13 15 17 19 21 23 25 27 29 31 33 35
Pit Number
Adjusted Metal 90%
Kriged Metal
Adjusted Metal 110%


Discounted Cash Flow Analysis
Figure 2.
Discounted Cash Flow
Millions
450
400
350
300
250
200
1 3 5 7 9 11 13 15 17 19 21 23 25 27 29 31 33 35
Pit Number
Kriged Metal
Sim_1
Sim_2
Sim_3
Sim_4
Sim_5


Mill Feed Release
Figure 3.
Tons per Day Released
9500
9000
8500
8000
7500
7000
1 3 5 7 9 11 13 15 17 19 21 23 25 27 29 31 33 35
Pit Number
Adjusted Metal 90%
Kriged Metal
Adjusted Metal 110%


Mill Feed Release
Figure 4.
Tons per Day Released
12000
11500
11000
10500
10000
9500
9000
8500
8000
7500
7000
1 3 5 7 9 11 13 15 17 19 21 23 25 27 29 31 33 35
Pit Number
Kriged Metal
Sim_1
Sim_2
Sim_3
Sim_4
Sim_5


Average Grade of Mill Feed
Figure 5.
0.045
0.04
0.005
0.00
Mill Feed Grade
1 3 5 7 9 11 13 15 17 19 21 23 25 27 29 31 33 35
Pit Number
Adjusted Metal 90%
Kriged Metal
Adjusted Metal 110%

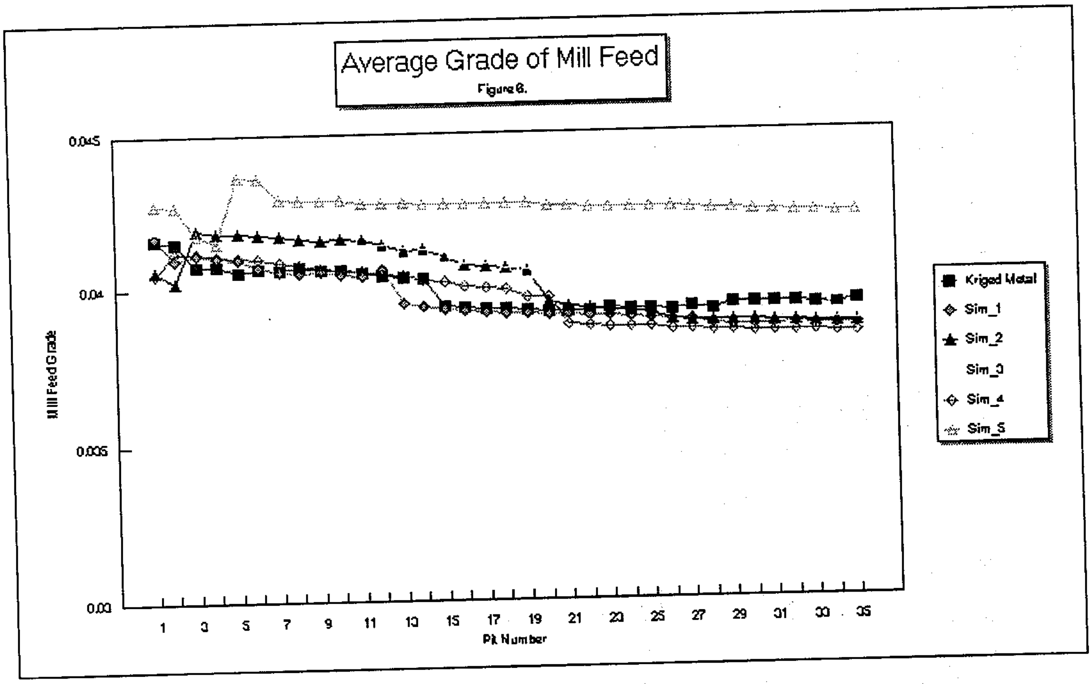

Average Grade of Mill Feed
Figure 6.
0.045
0.04
0.005
0.00
Mill Feed Grade
1 3 5 7 9 11 13 15 17 19 21 23 25 27 29 31 33 35
Pit Number
Kriged Metal
Sim_1
Sim_2
Sim_3
Sim_4
Sim_5

Table 1. Summary of Optimum Pit Scenarios By Process

Scenario	Total Tons	Waste Tons	S/R	DPAD Tons	DPAD Grade	DPAD Ounces	RPAD Tons	RPAD Grade	RPAD Ounces	Mill Tons	Mill Grade	Mill Ounces	Total Ounces Recovered	NPV @ 10%
Kriged Grade 90%	541,391,849	285,575,005	1.12	141,105,837	0.0139	878,027	84,577,929	0.0359	1,958,471	30,133,078	0.039	988,303	3,824,801	399,183,851
Kriged Grade	597,319,489	327,277,917	1.21	150,552,044	0.0139	926,137	87,298,702	0.0359	2,019,740	32,190,826	0.039	1,052,609	3,998,486	394,613,221
Kriged Grade 110%	711,011,814	416,912,498	1.42	166,469,233	0.0136	993,131	89,199,620	0.0350	2,054,794	38,430,463	0.039	1,261,439	4,309,364	386,711,887
K.G 90% vs K.G.	91%	87%	92%	94%	100%	95%	97%	100%	97%	94%	100%	94%	96%	101%
K.G. 110% vs K.G.	119%	127%	117%	111%	97%	107%	102%	97%	102%	119%	100%	120%	108%	98%
Sim_1	532,289,451	258,806,762	0.95	165,371,379	0.0129	820,920	69,755,126	0.0340	1,534,694	38,356,184	0.039	1,249,052	3,604,666	343,404,581
Sim_2	554,407,105	248,518,432	0.81	196,588,388	0.0129	1,059,265	74,677,286	0.0335	1,658,731	34,622,999	0.039	1,129,996	3,847,992	365,368,725
Sim_3	536,227,286	229,160,904	0.75	193,368,241	0.0129	1,050,171	75,523,368	0.0344	1,693,066	38,174,773	0.040	1,277,913	4,021,150	397,380,534
Sim_4	544,884,978	231,572,315	0.74	194,523,810	0.0130	1,065,696	81,117,365	0.0336	1,798,502	37,671,488	0.039	1,223,259	4,087,457	395,420,346
Sim_5	551,461,455	246,310,128	0.81	194,838,168	0.0129	1,050,469	71,801,776	0.0336	1,586,195	38,511,383	0.042	1,371,531	4,008,195	387,734,406
Sim_Min	532,289,451	229,160,904	0.74	165,371,379	0.0129	820,920	69,755,126	0.0335	1,534,694	34,622,999	0.039	1,129,996	3,485,610	343,404,581
Sim_Avg	543,854,055	242,873,708	0.81	188,937,997	0.0129	1,009,304	74,574,984	0.0338	1,654,238	37,467,365	0.040	1,250,350	3,913,892	377,861,718
Sim_Max	554,407,105	258,806,762	0.95	196,588,388	0.0130	1,065,696	81,117,365	0.0344	1,798,502	38,511,383	0.042	1,371,531	4,235,729	397,380,534
Sim_Min vs Krig	89%	70%	61%	110%	92%	89%	80%	93%	76%	108%	98%	107%	87%	87%
Sim_Avg vs Krig	91%	74%	67%	125%	93%	109%	85%	94%	82%	116%	101%	119%	98%	96%
Sim_Max vs Krig	93%	79%	78%	131%	93%	115%	93%	96%	89%	120%	108%	130%	106%	101%