
Conditional Simulations: Tools for Modelling Uncertainty in Open Pit Optimisation

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Abstract

Conditional simulation is a class of Monte Carlo techniques that can be used to generate equally probable representations of *in-situ* orebody variability. Contrary to the traditional smooth orebody models, conditionally simulated orebodies provide the tools to address uncertainty in grade variability and the resulting effects on various aspects of open pit design and production scheduling. This paper outlines a general framework for modelling uncertainty and assessing geological risk, presents currently used geostatistical simulation algorithms, and presents examples.

Introduction

Effective open pit design and production scheduling (OPDPS) is critical in surface mining ventures. The effects of pit design and scheduling and related predictions have major consequences for the management of cash flows, which are typically in the order of millions of dollars. Modern OPDPS is based on the well known Lerchs-Grossmann algorithm (Lerchs and Grossmann, 1965) implemented as the Nested Lerchs-Grossmann algorithm (Whittle, 1988, 1997; Whittle and Rozman, 1991). The latter algorithm provides an optimal scenario of how an orebody should be mined to best economical advantage, given a set of geological, mining and economic considerations. This optimal scenario is sensitive to the uncertainties related to the input to the optimisation process and, specifically, the uncertainties of (i) the orebody model and related *in-situ* grade variability and material type distribution, (ii) technical mining specifications such as slope considerations, excavation capacities, etc., and (iii) economics including capital and operating costs, and commodity prices. Various issues of uncertainty and risk involved in OPDPS have been raised in recent years (e.g. Ravenscroft, 1992; Onur and

Dowd, 1993; Halatchev and Moustakerov, 1994; Dowd, 1994; Denby and Schofield, 1995; Rossi and Van Brunt, 1997; Quayle and Cutts, 1997).

Orebody models and their geological characteristics are widely acknowledged as a major source of uncertainty and risk. In most cases, sensitivity to grade variability or metal values is tested with global changes (Whittle, 1993) which, however, can not account for the critical local block grade variability. In traditional orebody modelling (David, 1977, 1988), the question of block grade uncertainty may be addressed in terms of the estimation variance or an estimate of the expected distribution of block grades. Unfortunately, OPDPS is a non-linear process, thus any confidence intervals derived directly from block models may not be appropriate for optimisation purposes. Furthermore, the non-linear nature of optimisation processes suggests caution in analyzing predictions from OPDPS based on single smooth, "average" type orebody model, as they do not necessarily represent 'average' type predictions.

To address the issues of orebody uncertainty as they link to OPDPS, an alternative to the traditional orebody modelling methodologies is proposed using stochastic conditional simulation (Journel and Huijbregts, 1978, 1992; David, 1988; Dimitrakopoulos, 1990, 1994; Armstrong and Dowd, 1994; Baafi and Schofield, 1997; and others). Conditional simulation is a class of Monte Carlo techniques (Halton, 1970) that can be used to generate equally probable representations of the *in-situ* orebody grade and material type variability.

In the following sections, a general framework for dealing with geological uncertainty in open pit design and scheduling is first presented and examples are given. Subsequently, three main geostatistical simulation algorithms are presented and their characteristics are discussed. Effects on

OPDPS are then considered and suggestions are made.

A framework for modelling orebody uncertainty and examples

This section outlines a general framework for dealing with the uncertainty of geological parameters used for OPDPS. The framework includes three parts: (i) stochastic conditional simulations; (ii) transfer functions; and (iii) uncertainty modelling and risk assessment.

Modelling uncertainty associated with a mineral deposit

Drilling data represent initial information about a given mineral deposit and are used for deposit block

modeling. Block models are subsequently used as input to open pit optimisation. The actual deposit is partially known and the properties of interest such as grades and ore material types are inferred. In dealing with the unknown deposit and its attributes of interest, one may generate several models (images) of the deposit based on and conditional to the same data and statistical properties (Figure 1). These models represent the same deposit and are all constrained to (a) reproducing all available information, and (b) being equally probable representations of the actual deposit.

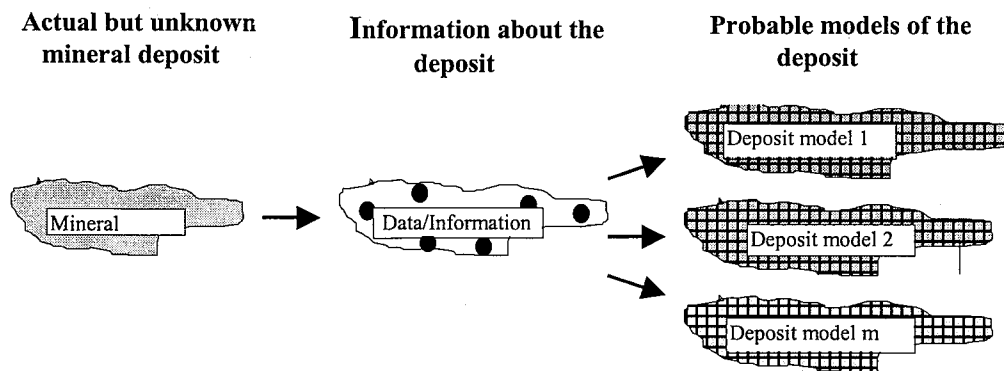


Figure 1: Describing uncertainty about a mineral deposit.

A series of simulated models of the deposit can represent or capture the uncertainty about the actual description of the deposit. Figure 1 schematically illustrates the idea. Two questions follow: (a) how to simulate equally probable models of the deposit; and (b) how these models can be used to solve specific mining problems. The first question is addressed in the general context of geostatistical or stochastic simulations (Journel and Huijebregts, 1978; Alabert, 1987; Dimitrakopoulos, 1990; Verly, 1992; Dietrich, 1993; Journel, 1994; Deutsch, 1994; and others). The second question is addressed here in the general context of transfer functions and optimal decision making.

Transfer functions and the modelling of a mining process

A mining process or a sequence of processes, such as open pit design and production scheduling is conceptualized here as a transfer function. The

specification of a transfer function depends on the problem under consideration. For example, in pit optimisation, Whittle 4D is a specific transfer function used in the mining industry. Other examples of transfer functions include the grade control and ore classification in a gold mine (e.g. Glacken, 1997), additional drilling programs, recoverable reserves, short term scheduling, stope design, and others.

Mining processes or transfer functions have specific parameters of interest for analysis or optimisation, for example, OPDPS includes parameters such as the maximum NPV pit shell, discounted cash flow, average mill feed grade, recoverable reserves. The output of a transfer function providing the parameter of interest to be analysed and assessed, and upon which decisions will be made is termed a response parameter. Examples of response parameters may include a series of dollar values of possible

ore/waste dig lines for ore control, or, in the case of pit optimisation, series of pit shells, NVP changes, tonnage to the mill feed, and so on.

Modelling uncertainty about the response

For a given possible description of a mineral deposit, a set of possible values for a parameter of interest may be selected. A computerised mining process such as OPDPS can then be applied for each of the selected values. Depending on the mining process at hand, the selection of parameters of interest can be formulated as an optimisation

problem where the objective is to maximise profitability.

For a set of possible deposit models and each value(s) for the parameter(s) of interest, the transfer function will generate a sequence of distributions of responses that can be seen as a map of the space of uncertainty of the response. If this mapping is adequate, then the optimization function will provide the expected results in terms of possible outcomes, ranges of expected values and optimal choices.

Figure 2 schematically presents the various parts of the methodology suggested.

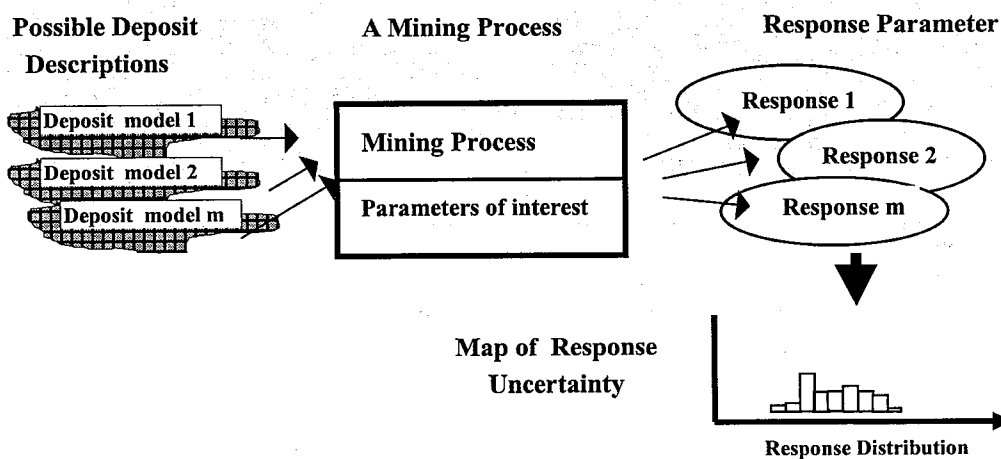


Figure 2: Diagrammatic representation of the proposed framework

Two important points

As noted earlier, mining transfer functions are generally non-linear. As a consequence, (i) an average type block model may not provide an average map of the space of response uncertainty; and (ii) a criterion for generating deposit descriptions may be defined as follows. The simulation technique selected for modelling must be evaluated in terms of the mapping of the response uncertainty.

Point (i) above suggests caution when analysing the results of a study involving complex mining processes and predictions such as OPDPS. Although predictions can in practice be reasonable, they do not necessarily capture all aspects of the uncertainties related to the orebody. Point (ii)

indicates that one may study the specific effects that simulation algorithms may have to the specific parameters of interest in OPDPS and select those that provide adequate mapping of the related space of uncertainty.

Some examples

The above methodology is demonstrated using a uranium deposit. Figure 3 shows four equally probable models of accumulation (the product of thickness and grade) all based on the same exploration drilling data. Figure 4 presents the results of the simple transfer function 'areas for additional drilling', constructed from the combination of 100 simulations of the deposit. The black areas on the block model indicate the parts with a greater than 80% chance that accumulation is

above an economic cut-off. Light gray outlines areas with an over 80% chance that accumulation is below the economic cutoff. The intermediate gray

areas are those where additional drilling would be most likely to provide useful information.

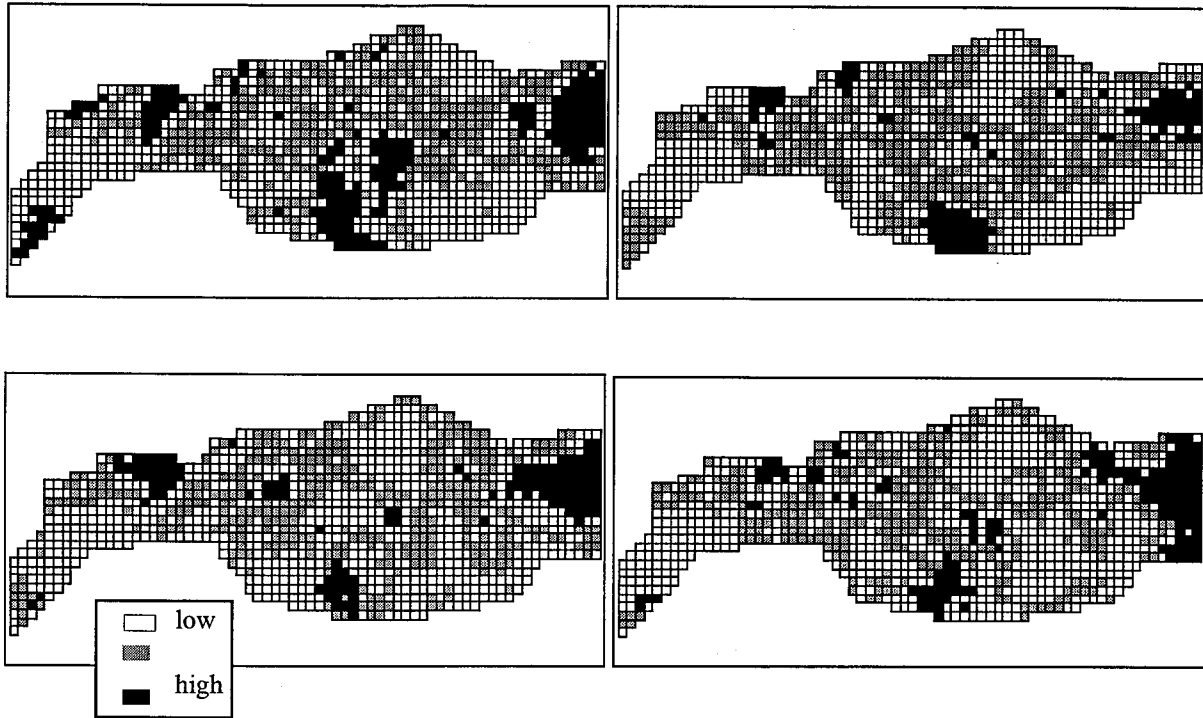


Figure 3: Simulations of accumulation in an uranium deposit based on exploration data.

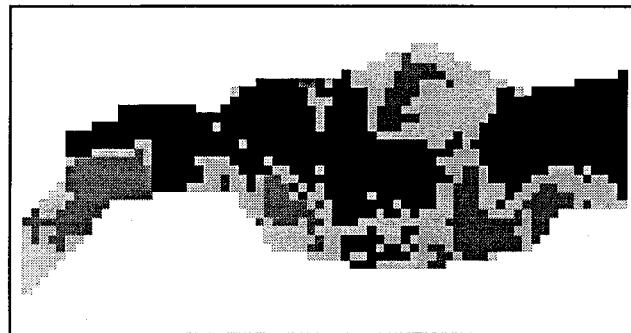


Figure 4: Identifying areas for additional drilling (dark gray); see text for details.

An example of the use of a more complex transfer function and the resulting map of response uncertainty is presented in Table 1. The table reports the results of the most profitable dig line (dig line 6) in a part of a bench in a pit, based on 50 simulations of gold grades from blasthole data. The diglines are generated using a floating cone-like algorithm that draws ore outlines based on mining and economic criteria. The combination of dig lines and simulated grades generated the results, some of which are shown in Table 1.

Dig line 6	Value: 9648.935
Dig line 25	Value: 9253.273
Dig line 12	Value: 7586.481
•	•
•	•

Table 1: ‘Equivalent dollar’ value for ore/waste dig lines in a bench of a gold deposit. The highest dollar value corresponds to the dig line that yields maximum profit.

Figure 5 shows a ‘quick and dirty’ example of an attempt to assess the effects of the orebody description on Whittle 4D (Whittle, 1993). In this example, a run that uses a smoothed orebody from the Whittle 4D manual (run 1) is shown together with a run where the orebody is ‘de-smoothed’ using conditional simulation (run 2). While this example is too limited to substantiate conclusions, it demonstrates some effects. For example, Figure 5 may suggest that, when the de-smoothed orebody model is used, the starting point for design is no longer pit 26 but pit 25.

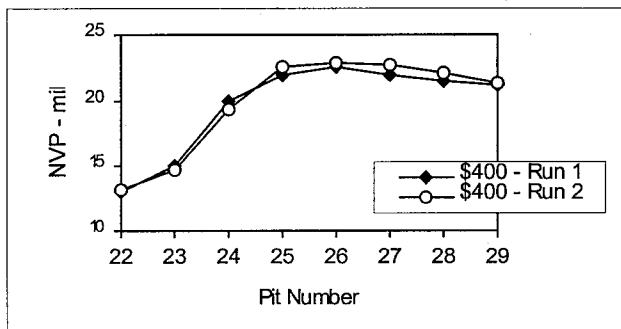


Figure 5: NVP versus pit shell for a smoothed orebody (run 1) and a corresponding simulated one (run 2) for a \$400/oz gold price.

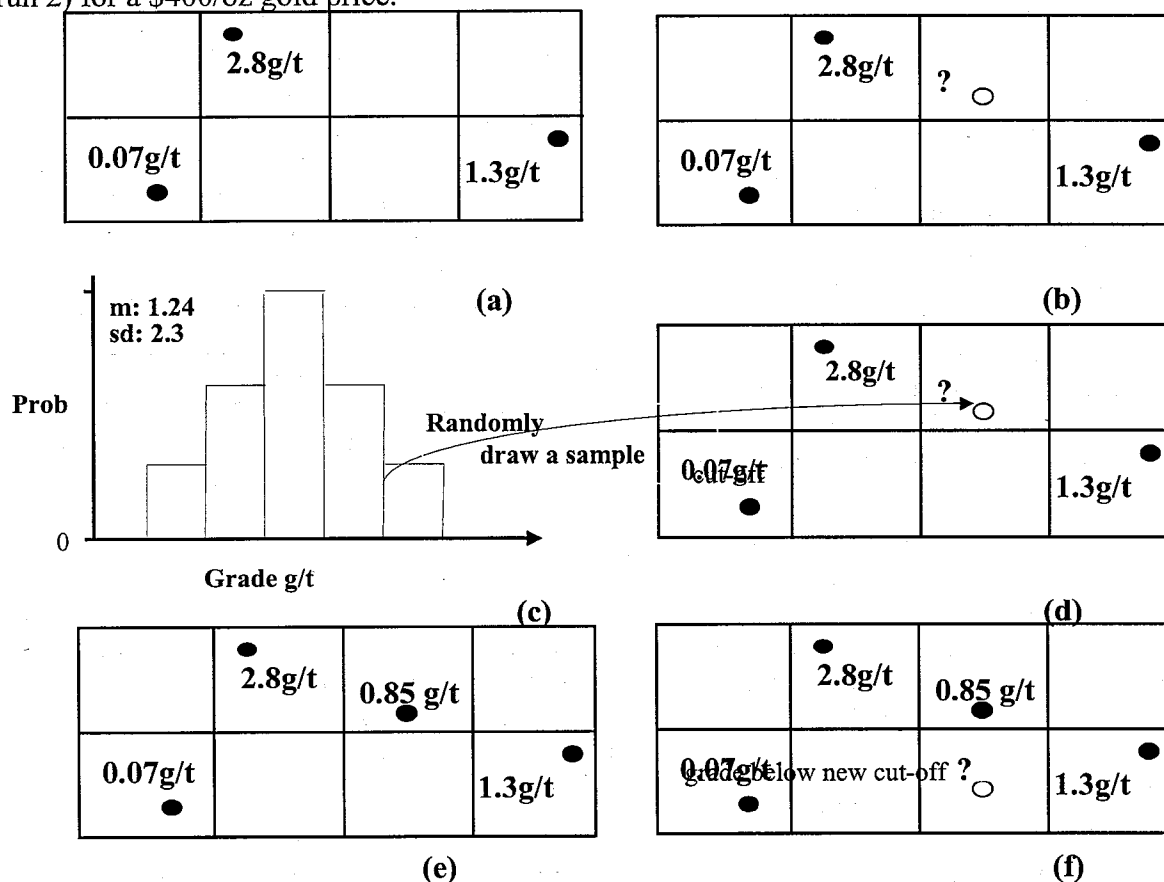


Figure 6: Sequential simulation of gold grades in a deposit: (a) deposit and data; (b) a location to simulate; (c) estimated the local conditional probability distribution (lcpd) at the location in (b); (d) random drawing from the lcpd; (e) adding the simulated value to data; and (f) repeat at a new location.

Conditional simulation algorithms

This section outlines the main geostatistical simulation algorithms that are currently in common use. Following the outlines, examples and comments on practical implementation, advantages and disadvantages are discussed.

Sequential conditional simulations

Sequential simulations are a family of simulations based on the same basic algorithm. This basic algorithm, schematically presented in Figure 6, consists of the following steps:

1. Randomly select a grid node yet to be simulated
2. Estimate the local conditional probability distribution (lcpd) of the grades at that grid node
3. Randomly draw a grade value from the lcpd
4. Include the simulated value in the conditioning data set
5. Repeat points 1 to 4 until all grid nodes have a value
6. Repeat points 1 to 6 to generate additional equally probable models.

Two main variants of the sequential algorithm are used in practice. Sequential Gaussian simulation or SGS (Journel, 1994; Johnson, 1987) uses simple kriging to estimate the lcpd. This process assumes that the lcpd is the classical normal distribution with the mean and variance of the lcpd being equal to the simple kriging estimate and estimation variance respectively. Application of SGS requires normalisation of the data and back-transformation of the results.

A recent fast, sequential algorithm (Dimitrakopoulos and Luo, 1997a) is based on properties of the LU decomposition of the covariance matrix (Scheuer and Stoller, 1962) in the form of a sequential group Gaussian simulation. The major advantage of this algorithm is greater speed than the typical SGS algorithm.

The second implementation of a sequential simulation is based on the estimation of the local cpd using Indicator Kriging (IK), and it is termed Sequential Indicator Simulation or SIS (Alabert, 1987). Similarly to IK, SIS does not make any assumptions on the shape of the local cpd, which is explicitly estimated. As a result, SIS is slower and more tedious in terms of implementation when compared to SGS. Various indicator kriging

alternatives may be used to estimate the local cpds (e.g. Dimitrakopoulos and Dagbert, 1993).

Probability field simulation

In the previous section, the sequential algorithm used every simulated value together with the actual data set to estimate the lcpd at each grid node. In order to save computational time from re-estimating lcpd's, Probability Field Simulation (PFS) can be used. (Figure 7). PFS consists of the following steps:

1. Use original data to estimate the local cpd at all grid nodes
2. Create a non-conditional simulation (probability field) in $[0,1]$ using a scaled variogram of the original data
3. Retain the percentile corresponding to the probability field at each the grid node
4. Repeat point 2 and retrieve percentiles in point 3 to generate additional realisations.

The major difference between PFS and the sequential algorithms, as well as the reason for increased speed, is in estimation of the lcpds at the grid nodes. lcpds are estimated in a single step and are conditional only to the original data. The control over the final results and the reproduction of variograms comes from step 2, as neighboring locations will tend to have similar probability values. This is in contrast to the sequential methods where continuity is ensured by adding the simulated data to the actual data set. For step 2 of the PFS algorithm, any simulation algorithm can be used (Froidevaux, 1992), however, as speed is a concern SGS is a common choice.

Simulated annealing

Unlike sequential and probability field simulations, Simulated Annealing (SA) is a combinatorial optimisation algorithm based on an analogy to the physical process of annealing as known, for example, in metallurgy. SA consists of the following steps:

1. Establish a grid of blocks with values honouring the available data and their distribution

2. Define a global 'goodness' criterion (objective or 'energy' function) that minimises the mismatch of desired properties and the grid values
 3. Swap randomly several non-data grid values
 4. Retain the swap if the 'energy' function lowered, otherwise reject the swap and try another
 5. Reduce the number of swaps as the 'energy' function gets closer to the target
 6. If objective function close enough to desired value, stop the process.
- SA is commonly implemented on a simple variogram-based objective function, where the desired objective function is the difference between the model and observed variogram.

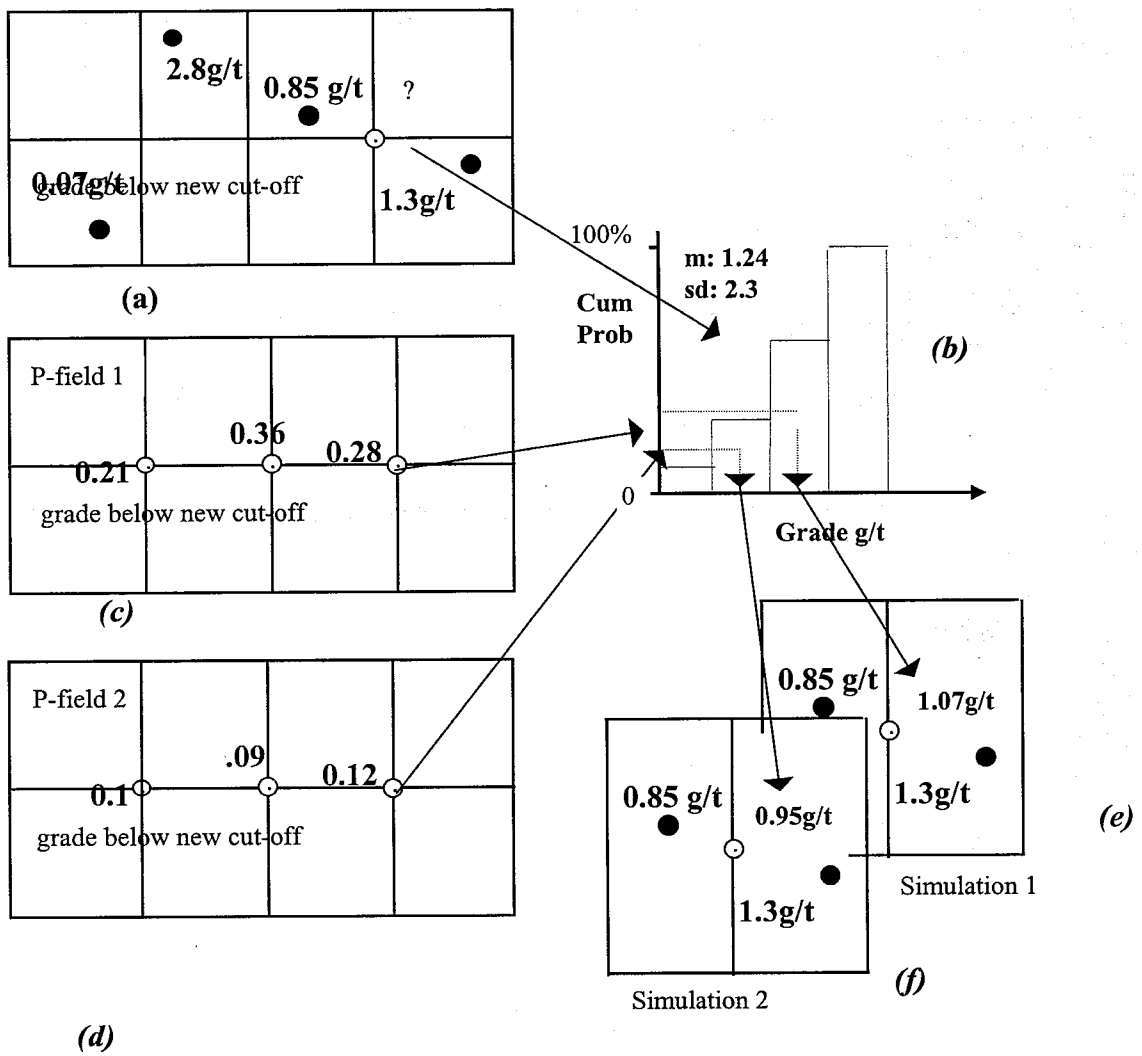


Figure 7: Probability field simulation of gold grades in a deposit: (a) deposit; (b) estimated lcpd at a grid node; (c) and (d) probability fields; (e) and (f) simulations at the grid node from the sampling of lcpd in (b) using (e) and (f).

However, the objective function can be generalised as a weighted average of several components such as variograms, indicator variograms, conditional distributions, and so on. SA is not as computationally intensive as it may appear (Deutsch, 1994; Deutsch and Journel, 1992). For instance, annealing implementations allow several swaps and then update the 'energy' function instead of recalculating it after each swap. An additional aspect of annealing is that in step 4, energy or the value of the objective function is allowed to increase based on criteria such as the Boltzman distribution, thus the tolerance differences and number of swaps are larger at early stages of annealing process and decrease as time passes.

The major advantage of annealing is flexibility in the reproduction of characteristics of interest. In this regard, mining applications may be improved by enhancing the spatial relations of grades when simulating a deposit. For instance, in addition to the reproduction of variograms (two point statistics), the annealing process can be formulated to impose the reproduction of multi-point statistics, which describe the connectivity of various ore grade categories. An additional advantage of SA is that it can use experimental statistics, such as smoothed experimental variograms (Dimitrakopoulos and Luo, 1997b) that may be more suitable in some applications.

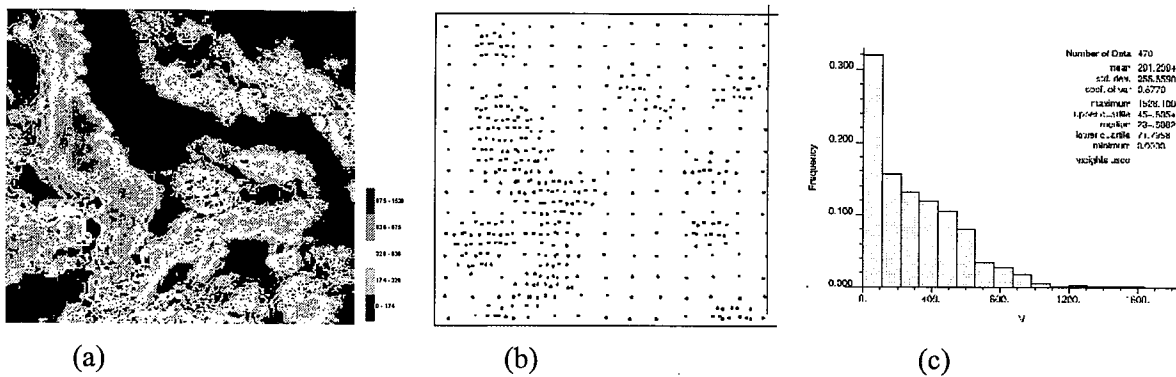


Figure 8: (a) Exhaustive data; (b) sample data locations with clusters in the areas of high values; (c) histogram of declustered sample data.

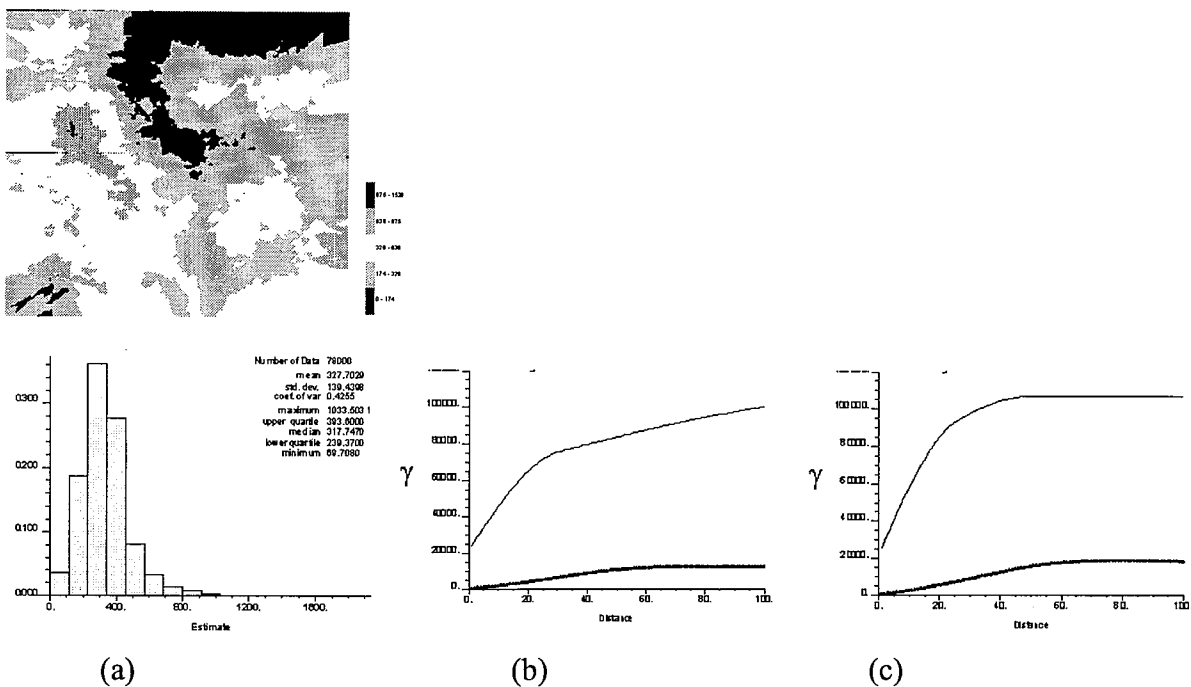


Figure 9: (a) An ordinary kriging block model of the data in Figure 8b; (b) histogram of the grades in (a); and (c) experimental variograms of grades in (a) and model used in kriging.

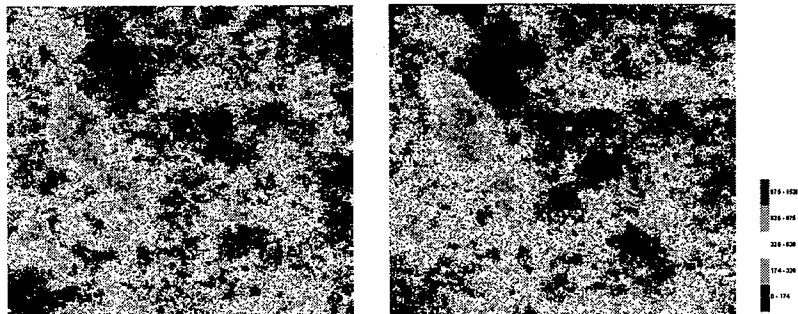


Figure 10: Two conditionally simulated models using the sample data and sequential Gaussian simulation algorithm.

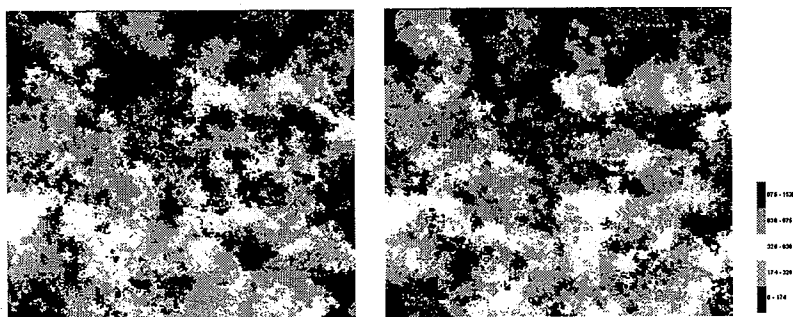


Figure 11: Two conditionally simulated models using the data in Figure 8b and the sequential indicator simulation algorithm.

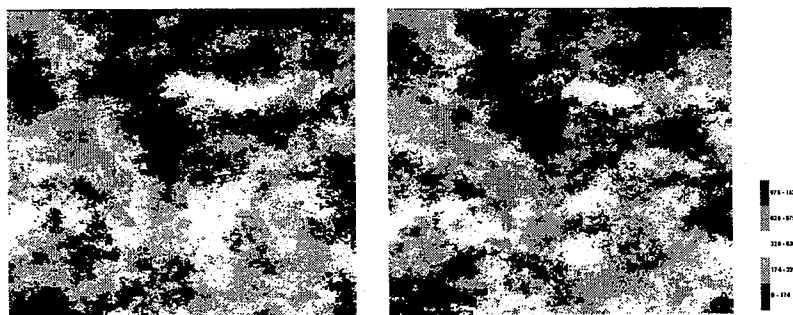


Figure 12: Two conditionally simulated models using the data in Figure 8b and the probability field simulation algorithm.

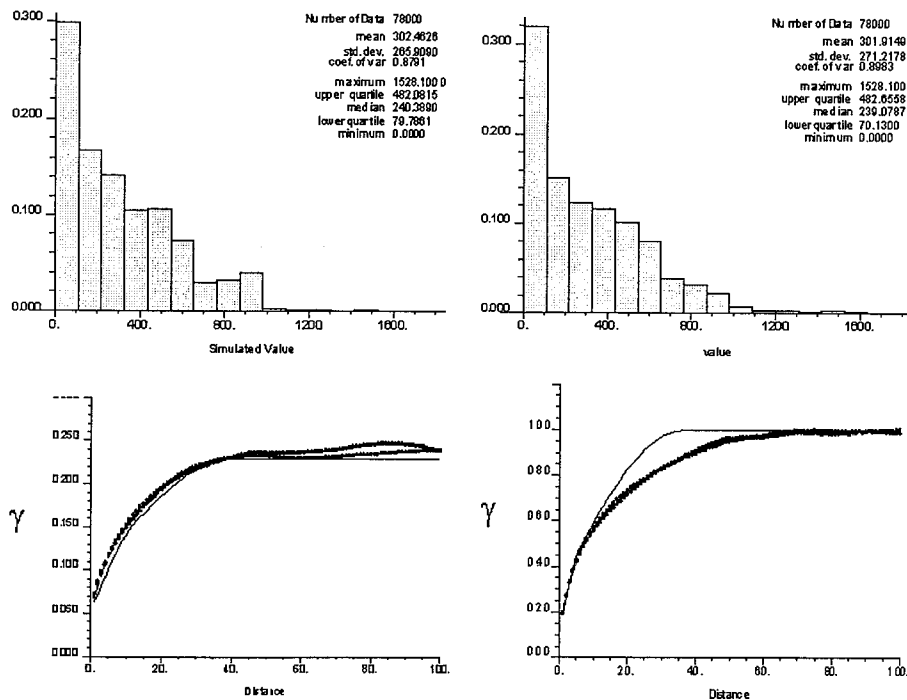


Figure 13: Sample histograms and variograms from different simulated realisations.

Examples and comparisons

To facilitate the communication of the some basic characteristics of conditional simulations, this section uses real, public domain, exhaustive data (Isaaks and Srivastava, 1989). The exhaustive data are shown in Figure 8a, while a sample data set derived from the complete data is presented in Figure 8b. The sample data set mimics mineral exploration data sets in that data are clustered in the areas of high values.

Figure 9 shows a block model based on the data set and ordinary kriging, a typical block modelling method, and the histogram and variograms of the estimated model. As expected, the corresponding histogram and variograms demonstrate the substantially lower variability of the kriged model compared to the actual data. This is typical for any smooth, 'average' type orebody model and is independent of the specific estimation technique used.

Figures 10, 11 and 12 show conditional simulations and associated statistics generated using the SGS, SIS and PFS algorithms. Examples showing the reproduction of statistics in simulated realisations

are given in Figure 13. The differences of the simulated realisations in the figures from the kriged block model are distinct. Although based only on 470 samples out of a total 78,000 in the exhaustive data set, the simulations better represent the features of the original data, while at the same time better reproduce spatial variability. It seems reasonable to suggest that the simulations in this example generate more plausible models than the estimation algorithms.

Conditional simulation and pit optimisation

Contrary to smooth estimated models, conditionally simulated orebody models are promising tools in describing and transferring geologic orebody uncertainty to pit design and scheduling. The paper by Rossi and Van Brunt, in this volume, provides a case study with an eloquent demonstration advocating the use of conditional simulations in conjunction with Whittle 4D.

Considering that conditional simulation algorithms are readily available, additional case studies are required to elucidate on the various aspects and effects of grade uncertainty to the various aspects of open pit optimisation and long term scheduling.

Comments and conclusions

The most common initial reactions to conditional simulations are surprise and skepticism. Surprise because (a) there are several equally probable models compared to the usual single model derived from the data, and (b) models look too complex compared to the smooth and well behaving orebody models traditionally used. Skepticism is created from the simple question: which one of the models is the correct one or which one should be used? However, when the complexities of both mineral deposits and optimisation process are considered, neither the models appear too complex any longer nor the availability of several equally possible models is a surprise. A simple inspection reveals that all simulated models share the same major features and reproduce the same data. The skepticism on which model to use reflects the common practice of constructing and using a single orebody model. This single representation of the orebody is considered as 'the average' or the 'best' that can be done with the available information. However, such skepticism about multiple models may easily be redirected to skepticism concerning the use of a single orebody model and the possible effects on evaluating, predicting, designing, planning and producing.

Conditional simulation methods are presented here as complementary tools to Whittle 4D, focusing on modelling the geological uncertainty in pit optimisation and the related effects on optimisation, predictions, scheduling and planning. Several simulation algorithms such as the ones described in this paper are readily available and can be used in optimisation studies. Which algorithm to use depends on many factors. Availability of reliable and well tested routines is a first criterion. The use of SGS or SIS or PFS also depends on the efficiency of specific implementations and the hardware that is available. Generally, SGS is simpler and tractable, but is controlled by its Gaussian nature. SIS is flexible conceptually, but it is relatively slow and space demanding. PFS is fast and efficient, but perhaps less readily available. Simulated annealing is powerful, and promising but also computationally intensive and, perhaps, less well understood.

The methodology presented in this paper for dealing with uncertainty is general and can be used in any mining process that links to the mined orebody. The methodology may be seen as a 'top down' approach

which considers mining parameters of interest and tests the effects of the parameter values and map the possible parameter values. In this sense, the major criterion for the orebody models used is how adequate are they in providing a map of the possible mining parameter values.

The practice of conditional simulations requires more time, effort and expertise compared to traditional approaches. However, their ability and promise to deal with geological uncertainty in open pit design and scheduling suggests that their use is a well worth the effort.

Acknowledgments

Acknowledgments are in order to Marcelo C. Godoy for generating the Walkerlake examples.

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