Cut-off Grades Beyond the Mine - Optimising Mill Throughput

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Abstract

Cut-off grade optimisation is used to arrive at an operating strategy that maximises the value of a mine. Where the mine’s capacity allows, sacrificing low-grade material enables the mill to process ore that delivers a higher cash flow.

A similar exercise may be undertaken to determine an optimum strategy for a mill. Reducing the residence time of ore results in a higher tonnage throughput but at a reduced recovery of mineral. In spite of this the net result can be an increase in the rate of mineral production and cash flow for the operation.

It follows from this parallel that tools such as Opti-Cut, although originally developed for determining the optimum cut-off grade strategy in the mine, might also be used to find a solution for the mill. The paper describes a model for the mill that enables cut-off optimisation to be applied.

The dependence of mine cut-off grade on mill capacity means that the optimum strategy for each cannot be determined in isolation. By using its multi-element features, Opti-Cut can be used to simultaneously optimise the mine cut-off grade and mill throughput.

Introduction

Optimising mine cut-off grades has long been recognised within Rio Tinto as a means to maximise the Net Present Value (NPV) of mining projects. From the work of Lane K F (1988) and others, the group pioneered the development of software in this field with the Optimum Grades for Resource Exploitation (OGRE) program.

In recent years the ideas behind mine cut-off optimisation have been extended to encompass other parts of the operation. Primarily this involves optimising the throughput and recovery in the mill.

A mill may be operated at a high throughput rate, sacrificing recovery but achieving a higher rate of production of mineral. This is equivalent to a mine operating at an above-breakeven cut-off grade, sacrificing some of the resource to obtain a higher cash flow. This parallel suggests that tools developed for determining the optimum cut-off strategy in the mine might also be used to find a solution for the mill.

Because the mine and mill are interdependent one cannot be optimised independently of the other. Past practice has been to optimise the mine assuming a fixed mill capacity and recovery. Studies
are repeated at a range of mill recovery/throughput settings to obtain an optimum. In certain conditions this can produce an acceptable result.

The paper describes the simultaneous optimisation of the mine cut-off grade and mill throughput. It shows the parallels between the modelling of the recovery of material in a mill and the recovery of a resource in the mine. The benefits are demonstrated with the results from optimising two cases.

The method utilises two-dimensional multi-element optimisation. Techniques for building models and utilising Whittle Programming’s Opti-Cut are described in an appendix.

**The Mill Capacity / Cut-off Grade Relationship**

Ore is ground in a mill to liberate mineral from the surrounding rock matrix. Recovery of mineral in the subsequent flotation circuit increases with increasing mill residence time. However there is a point beyond which the incremental value gained is exceeded by the additional costs incurred. Where feed to the mill is constrained by the mine’s capacity this is the optimum operating point and is equivalent to working to a breakeven cut-off grade in the mine.

If the feed is unconstrained the mill may be operated at a higher throughput rate, sacrificing some recovery but achieving a higher overall production of mineral. This is synonymous with a mine operating at an above-breakeven cut-off grade, sacrificing some of the resource to obtain a higher cash flow.

Figure 1 illustrates a set of mill characteristics for a hypothetical mill. It shows the recovery in the mill and the capacity at various operating settings (mill configurations). Description of the techniques for deriving these characteristics is outside the scope of this paper.

Configuration 1 represents the point that maximises recovery in the mill. From left

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**Figure 1: Mill Operating Characteristics**
to right the higher numbered configurations represent points of coarser grind and increased tonnage throughput. Along with this increase recovery declines.

The inverse of mill throughput is modelled as mill consumption (shown as days / million tonnes of ore for clarity). Moving to a coarser grind (increasing mill configuration) consumes less time per unit of ore. In a given period more ore can be processed and, in spite of the lower recovery, more mineral produced. This has a parallel in the mine where a higher cut-off grade results in a higher throughput of the resource, a higher rate of mineral production, but at a lower recovery of mineral from the resource.

Note from Figure 1 that the trend of the mill consumption curve follows the trend of a cumulative ore tonnage vs cut-off grade curve (not illustrated). Likewise the recovery curve follows a cumulative product vs cut-off grade curve. Thus mill configuration\(^1\) has the characteristics of a cut-off grade, albeit a parametric one. Parametric cut-off grades are described by Lane K F (1988, Chapter 11).

Cut-off grade optimisation maximises the NPV of a project subject to capacity constraints in the mine, the mill and the market. These are usually expressed as annual limits to the tonnage mined, tonnage milled and product sold. At any point in time at least one limit, and possibly two or all three, will be constraining the system.

For cut-off optimisation to work correctly, capacity constraints must be independent of the cut-off grade. Yet varying the mill configuration changes the tonnage capacity of the mill. It is not independent of the cut-off being optimised.

The author (Wooller, 1997) demonstrated that both mine and mill capacity constraints could be modelled as market constraints. In other words both ore and total mining quantities could be dealt with as products. This enables several facets of a mining operation to be modelled as capacity constraints; not just those measured in terms of rock mass.

In particular it was shown that a mill’s annual capacity could be expressed in time units (days). Unless there are changes in the operating efficiency this will be constant. The consumption of this time is the inverse of the mill throughput and can be expressed as a grade (days/tonne). This enables the mill capacity to be modelled as a market limit, which is independent of the mill configuration (cut-off grade). Its consumption is a product (days) whose quantity will vary with the mill configuration but whose total will be subject to the mill capacity limit.

For an increment of the resource fed to the mill the mineral grade will be constant. Its recovery will vary with the mill configuration. Multiplying it by the recovery at the various mill configurations generates a recovered grade curve that retains the cumulative properties suitable for use in cut-off optimising.

Having shown that the mill configuration can be treated as a cut-off grade the next step was to construct a model to enable it to be optimised concurrently with the mine cut-off. Opti-Cut was chosen as it supports the simultaneous optimisation of two or more cut-offs. The techniques used in building the Opti-Cut model for the subsequent optimisations are described in the appendix.

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\(^1\) Several attributes were considered to define the mill cut-off including throughput, sieve size, P80 and mill configuration. Although not uniquely so, the mill configuration as described has the advantage of mirroring the behaviour of the mine cut-off.
Results from Optimisations
To demonstrate the application of the technique two scenarios are presented. The first is a system that is solely mill limited over the life of the operation, the second starts as being mine limited in its early life until sufficient ore is developed for the operation to become mill limited. The solely mine limited case is not presented as the best option is always to maximise the recovery in the mill.

![Graph showing present value and life vs mill throughput for constant and variable throughput scenarios.](image)

**Figure 2: Mill Limited Case: Constant vs Variable Throughput**

A high cut-off grade policy will usually be operated in conjunction with stockpiles, which can have an impact on the optimum cut-off grade policy. These trials were run without stockpiles.

**Mill Limited Case**
Ten optimisation runs were performed; the first nine each being run at a fixed mill configuration for the life of the mine. In the tenth Opti-Cut was run using a technique to optimise simultaneously the mine cut-off and mill configuration. The results are shown in Figure 2.

This clearly shows the advantage of operating the mill at a higher throughput/lower recovery when the operation is solely mill constrained. Most of the improvement in value is gained by moving from the point of lowest throughput/maximum recovery.

At about 12.4 Mt/y there is a mill configuration that, if set for the life of the mine, gives a present value (PV) close to that obtained by performing a simultaneous mine/mill optimisation; although the latter performs marginally better.

Figure 3 shows the mine cut-off, mill configuration and throughput from the simultaneous mine/mill optimisation.

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2 Stockpile material comes from that rejected as being below cut-off, in this case the unshaded quadrants in Figure 6. As this includes mineral that reports as tailings in the mill, the grade of the stockpile is overstated. The ideal case would be to include only that material represented by the unshaded quadrants in Figure 7. This might require an enhancement to Opti-Cut.
The mine cut-off characteristically declines over the life of the project, as also does the mill configuration (or cut-off). This is reflected in the higher mill throughput in the early years, declining for the last two years of full production.

**Mine/Mill Limited Case**
In this scenario the operation is mine limited during its early years, the same set of ten optimisations being run.

Figure 4 compares the PVs at various fixed mill configuration settings with that resulting from the simultaneous mine/mill optimisation.
Again there is a single fixed mill configuration that delivers a maximum PV. It occurs at a lower mill throughput than the mill limited case. However simultaneously optimising the mine and mill gives a significantly better result.

Figure 5 shows the mine cut-off, mill configuration and throughput from the simultaneous mine/mill optimisation in the mine/mill limited case.

![Graph of Cut-off Grade vs Year and Mill Throughput vs Year](image)

**Figure 5: Mine/Mill Limited Case: Cut-Off vs Mill Throughput**

The mine cut-off is low in the early years when the operation is mine limited, rising as the operation moves from being mine to mill limited. It then declines, following the trend shown in the mill-limited case above.

The mill configuration, and therefore the mill throughput, follows the same trend. At times it drops to the lowest practical setting, i.e. that which maximises the recovery of mineral in the mill. In Figure 5 these points are off the chart.

**Conclusions**

It is possible to model the behaviour of ore in a mill in a way that allows its optimum configuration to be determined using mine cut-off grade optimising software. A model can be constructed which allows both the mine cut-off grade and mill configuration to be optimised concurrently, reflecting their interdependence. The technique can be implemented using the multi-element features available in Opt-Cut. This can lead to a higher NPV for an operation than can be achieved by optimising the mine and mill in isolation.

**Acknowledgements**

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**References**


Modelling the Mine/Mill Relationship in Opti-Cut

For simplicity a single mineral operation has been chosen as an example, in this case a copper mine. In the following set of figures the mine (copper) cut-off is shown along the vertical axis and the mill configuration along the horizontal axis. For material to be processed and mineral recovered it must satisfy both cut-off criteria.$^3$

![Diagram of Mine/Mill Relationship]

*Figure 6: Recovery of Mineral from Mine and Mill*

In Figure 6 the shaded area represents the mineral above both the mine and mill cut-offs for an increment of the resource. As the mine cut-off grade is raised less of the mineral contained in the increment is recovered. Likewise as the mill moves from a fine grind to a coarse grind, synonymous with raising the mill cut-off, less mineral is recovered.

**Mill Feed**

Normally in a two mineral cut-off optimisation the shaded area in Figure 6 would also represent the ore fed to the mill. Material outside of this area would be sent to the waste dump or low grade stockpiles. This does not apply in the mine/mill case.

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$^3$ Cut-off mode 2 (CUM=2 in the Opti-Cut Economics Text File) must be used. This ensures that only material greater or equal to both the mine and mill cut-off is selected for processing. Although included in the Beta releases, cut-off mode 2 is not mentioned in the manual for Revision 1.00. However it has been retained in the Opti-Cut code.
In Figure 7 the shaded area represents the material fed to the mill in a mine/mill optimisation. It is independent of the mill cut-off. Unless the model used by Opti-Cut is modified the material in the upper left quadrant will not be included in the tonnage of material reporting to the mill and the optimisation will not work.

**Figure 7:** Mill Feed vs Mine and Mill Cut-offs

To overcome this in mine/mill optimisation potential mill feed is modelled as a product and placed in the distribution in the highest mill configuration cut-off interval for each mine (copper) cut-off interval. By capping the mill cut-off below this interval the potential mill feed will be unaffected by the mill configuration. The ore/waste split is determined solely by the mine cut-off as illustrated in Figure 8.

**Figure 8:** Mill Feed vs Mine and Mill Cut-offs (Sequence File)
The shaded area in Figure 8 represents the total potential ore. The portion above the mine cut-off is sent to the mill; the portion below is rejected as waste. Because the mill cut-off is capped, only the position of the mine cut-off determines the ore/waste split.

The following extract is from the Sequence Text File and illustrates the modelling of the (potential) mill tonnage as a product. The model groups have been created at intervals of 0.02% copper (CU_GRADE) and over a range of mill configurations from low throughput/high recovery to high throughput/low recovery.

<table>
<thead>
<tr>
<th>GR. ORE</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>ELR CU_GRADE</td>
<td>0.600</td>
</tr>
<tr>
<td>ELR MILL_CFG</td>
<td>67.00</td>
</tr>
<tr>
<td></td>
<td></td>
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<tr>
<td>ELA MILL_KTS</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>GR. ORE</td>
<td>1</td>
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<tr>
<td>ELR CU_GRADE</td>
<td>0.600</td>
</tr>
<tr>
<td>ELR MILL_CFG</td>
<td>70.00</td>
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<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>ELA MILL_KTS</td>
<td>88.2</td>
</tr>
</tbody>
</table>

The two groups represent the uppermost mill configuration intervals in the copper cut-off interval from 0.6% to 0.62%. The first is typical of all the mill configuration intervals from 1 to 70. In them the element MILL_KTS is zero. In the highest group MILL_KTS represents the tonnage of ore in the increment between 0.6% and 0.62% copper.

In building a 2-D model when both cut-offs are in the mine the group tonnage represents the material delimited by both sets of cut-off intervals. With one cut-off in the mill the incremental tonnage between mill configurations is zero. As product quantities are calculated from

\[ \text{product} = \text{tonnes} \times \text{grade} \times \text{recovery} \]

a problem arises in modelling the grades of the incremental mineral recovered between mill configurations.

The solution is to set the tonnage of material in all groups to unity (1.0), irrespective of whether or not they contain any material. The recoverable product quantity is used for the grade and the recovery set to 100%. The calculation of product quantities becomes

\[ \text{product} = 1.0 \times \text{product} \times 100\% \]

and zero grades are used when there is no material in an interval.

Although the tonnage of the groups will contribute to the economics through the mining cost, this is set to a low value and the effect is insignificant.
Total Mining

Figure 9 represents the material that makes up total mining. It is derived from the total tonnage from all groups, together with pure waste, and is independent of both cut-off grades. However, having set the tonnage for each group to 1.0 this cannot be used to derive the total mined tonnage.

![Diagram of Total Mining vs Mine and Mill Cut-offs](image)

**Figure 9: Total Mining vs Mine and Mill Cut-offs**

The solution is to model the mining tonnage as a product. Using the same logic as for mill feed, the total material in the increment, except that defined as pure waste, is placed in a single group in the highest mine cut-off and mill configuration interval. By capping the mine cut-off below this interval, in addition to the mill configuration cap, its quantity is independent of both cut-offs. This is illustrated in Figure 10.

![Diagram of Total Mining vs Mine and Mill Cut-offs (Sequence File)](image)

**Figure 10: Total Mining vs Mine and Mill Cut-offs (Sequence File)**
The following extract from the Sequence Text File illustrates the modelling of the mine tonnage as a product.

```
GR ORE 1
ELA CU_GRADE 2.116
ELR MILL_CFG 67.00 68.50 70.00
.

ELA MILL_KTS 0.0
ELA MINE_KTS 0.0
GR ORE 1
ELA CU_GRADE 2.116
ELR MILL_CFG 70.00 71.50 73.00
.

ELA MILL_KTS 3735.0
ELA MINE_KTS 12534.6
```

The two groups represent the uppermost mill configuration intervals in the highest copper cut-off interval. The first is typical of all the mill configuration intervals from 1 to 70. In them the element MINE_KTS is zero. In the highest group MINE_KTS represents the tonnage of all potential ore in the increment.

**Recoverable Copper**

The relationship of recoverable copper to the mine cut-off and mill configuration is illustrated in Figure 6. The recoverable copper within a copper cut-off range is distributed over the range of mill configuration intervals following the mill recovery curve from Figure 1. It is modelled as element COPPER. Units are the tonnage of recovered copper in the group.

```
GR ORE 1
ELA CU_GRADE 2.116
ELR MILL_CFG 67.00 68.50 70.00
.

ELA COPPER 4.94689
ELA MILL_KTS 0.0
ELA MINE_KTS 0.0
GR ORE 1
ELA CU_GRADE 2.116
ELR MILL_CFG 70.00 71.50 73.00
.

ELA COPPER 13.58081
ELA MILL_KTS 3735.0
ELA MINE_KTS 12534.6
```
Mill Hours

Included within each group in the model is the time taken to process it in the mill, expressed in hours as element MILL_HRS. These times have been derived from the curve shown in Figure 1.

<table>
<thead>
<tr>
<th>Group</th>
<th>MIL Configuration</th>
<th>Time (hrs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ORG</td>
<td>1</td>
<td>0.600</td>
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<tr>
<td>ELR</td>
<td>CU_GRADE</td>
<td>0.611</td>
</tr>
<tr>
<td>ELR</td>
<td>MILL_CFG</td>
<td>0.620</td>
</tr>
<tr>
<td>ELP</td>
<td>MILL_HRS</td>
<td>0.035</td>
</tr>
<tr>
<td>ELP</td>
<td>COPPER</td>
<td>0.03373</td>
</tr>
<tr>
<td>ELP</td>
<td>MILL_KTS</td>
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</tr>
<tr>
<td>ELP</td>
<td>MINE_KTS</td>
<td>0.0</td>
</tr>
<tr>
<td>ORG</td>
<td>1</td>
<td>0.600</td>
</tr>
<tr>
<td>ELR</td>
<td>CU_GRADE</td>
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</tr>
<tr>
<td>ELR</td>
<td>MILL_CFG</td>
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<tr>
<td>ELP</td>
<td>MILL_HRS</td>
<td>40.656</td>
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</tr>
<tr>
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<td>MILL_KTS</td>
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</tr>
<tr>
<td>ELP</td>
<td>MINE_KTS</td>
<td>0.0</td>
</tr>
<tr>
<td>ORG</td>
<td>1</td>
<td>2.116</td>
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<tr>
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<tr>
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</tr>
<tr>
<td>ELP</td>
<td>MINE_KTS</td>
<td>12534.6</td>
</tr>
</tbody>
</table>

In the example the time in the highest mill configuration for a given copper cut-off interval is the time required to obtain the minimum recovery. Dropping the mill cut-off, i.e. moving to a lower mill configuration, not only increases recovery of copper but also produces more MILL_HRS. As this is modelled as a product with a market limit the effect is to consume an additional increment of mill time.

Dropping the copper cut-off also increases the recovery of copper from the resource. As well as consuming more mill time, additional ore tonnage costs are incurred. This is, of course, offset against a lower mining cost per unit of ore as the waste:ore ratio is reduced.

Modelling Milling Costs

Because the available mill time is constant, the processing costs associated with MILL_HRS are fixed and are modelled in the fixed costs for the project. How mill costs are apportioned can affect the outcome of the optimisation.

Where the proportion of the fixed costs in the mill is high, the low incremental ore costs would tend to drive the optimiser to lower the mine cut-off rather than the mill in the search for more recoverable mineral. Where variable costs are relatively high, the trend will be to increase recovery in the mill by preferentially lowering the mill cut-off. Correctly identifying the balance between fixed and variable costs in the mill is crucial to obtaining the best result with this technique.
Economics Text File

The following is a listing of the Economics Text File used in the optimisations.

The run uses periods of 12 months, fixed costs of $50M/y (tonnage is measured in Ktonnes) and a discount rate of 10%. Included in the fixed costs is the fixed component of the mill costs.

ECO Example for Mine/Mill optimisation.
PL 12
TC A 50K
DI 10

The following are all defined as products but with no revenue.

PR CU_GRADE P 0.0
PR MILL_CFG P 0.0
PR MILL_HRS P 0.0
PR MINE_KTS P 0.0
PR MILL_KTS P 0.0

Copper sells at $2000/t with TCs/RCs and selling costs of $600/t.

PR COPPER P 2000.0
PR COPPER S 600.0

Mine cut-off uses the copper grade (CU_GRADE). The second cut-off is in the mill and uses the mill configuration (MILL_CFG).

EL CU_GRADE 2
EL MILL_CFG 1

The mining cost of $1.00/t is applied to waste.

RO WASTE M 1.000

Because mining is modelled as a product a token mining cost is applied to ore groups as each has been defined using a nominal unit rock tonnage. Opti-Cut does not accept zero as a cost.

RO ORE M 0.0000001

Mining ($1.00) and milling ($2.00/t) costs for ore are applied as a processing cost using the following formula\(^4\). Milling costs here represent only the variable component (tonnage dependent) costs in the mill.

MT MILL ORE (MINE_KTS.G*1.0)+(MILL_KTS.G*2.0)

As all quantities in the model are on a recovered basis, recoveries are set to 100%.

MTP CU_GRADE R 100
MTP MILL_CFG R 100
MTP MILL_HRS R 100
MTP COPPER R 100
MTP MILL_KTS R 100
MTP MINE_KTS R 100

The mine cut-off is capped at 1.2% to force all MINE_KTS to be recovered.

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\(^4\) By modelling them as elements the intention was to model mining and milling costs as selling costs for the products MINE_KTS and MILL_KTS. However Opti-Cut does not sell a parcel at a loss hence these costs were frequently not included in the optimisation.
MTC CU_GRADE A 1.2

The mill cut-off is capped at configuration 70 to ensure all MILL_KTS above the mill cut-off are recovered. In practice this is never reached.

MTC MILL_CFG A 70.0

To apply a mine limit potential ore mining and waste must be grouped.

TG TMINING MINE_KTS.Q+WASTE.Q
TL TMINING A 40K

Mill capacity is set at 8,000 hours/year.

TL MILL_HRS A 8K

Cut-off mode 2 must be used.

CUM 2